

The Missing Jolt:  
A Simple Refutation of the NIST-Bazant Collapse Hypothesis

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In its *Final Report on the Collapse of the World Trade Center Towers*, the National Institute of Standards and Technology summarizes its three year study and outlines its explanation of the total collapse of WTC 1 and WTC 2. [1]

Readers of the report will find that the roughly \$20 million expended on this effort have resulted in an explanation of the total collapse of these buildings that is so vague it barely qualifies as a hypothesis. But it does have one crucial feature of a hypothesis: it is, in principle, falsifiable. In fact, it is easy to demonstrate that it is false.

In this paper we will, concentrating on the North Tower, offer a refutation that is:

- easy to understand but reasonably precise
- capable of being stated briefly
- verifiable by any reader with average computer skills and a grasp of simple mathematics.

**NIST's Hypothesis of Total Collapse:**

Three essential elements of NIST's hypothesis of total collapse are made explicit in the Final Report and the companion volumes of the study:

1. Because of damage to stories 93 to 98, and especially because of column buckling due to fire, the top 12 stories of the North Tower (99-110) plus the roof were, in effect, separated from the rest of the Tower and began to behave as a unit. [2]
2. This "rigid block" of 12 stories plus the roof began to move. First it tilted, and then it abruptly fell onto the stories beneath it. [3]
3. The fall of the rigid block caused such damage to the lower structure that "global collapse began." [4]

The *rigidity* of the upper block of stories is crucial to this explanation. If the upper block were to break, disintegrate or flow on impact it would certainly not threaten the 92 intact floors beneath it.

In addition, the rigid block had to *fall* onto the rest of the building. Although this seems obvious, the NIST authors are often shy about saying it. We hear about the rigid block's "descent." [5] We hear of tilting and "downward movement." [6] We have to look carefully to find the NIST authors using the language of falling. Whatever the reasons for their reticence, it is clear that it will not do for the upper block to ease itself onto the building beneath it, with a gradual creaking of buckled columns and sagging floors. If this were to happen, why would the structure beneath collapse?

There was nothing special about the *weight* of the upper block, rigid or otherwise. The lower part of the Tower had held up this weight without difficulty since 1970. The lower block had 283 cold steel columns, with less than 30% of their total load capacity being utilized for gravity loads, because of the factors of safety designed into the structure and the need to withstand high winds—and gravity loads were essentially the only loads the columns would have been subject to on a day such as 9/11 with little wind. The lower block was not weak, nor (excluding stories 93-98) was it damaged by plane impact or fire. The weight of the upper block posed no threat to it. If there were to be a threat, it had to come from the *momentum* of the upper block. But momentum is a product of mass and velocity, and since the upper block could not increase its mass it had to increase, if it were to become a threat, its velocity. Since NIST's theory assumes the only energy at play at this stage of events was gravitational, the upper block had to fall, and the greater its velocity the greater its momentum. The longer and the less impeded its fall, the greater would be its impact on the lower structure. So it is no surprise that the NIST authors, however shy they are about affirming it, eventually come out in favour of the *falling* of the upper block. [7]

Zdenek Bazant and Yong Zhou, with whose September 13, 2001 back-of-the-envelope theory (with subsequent revisions and additions) NIST largely agrees, have never hesitated to say that the upper block fell. [8] Bazant has likewise been frank about the need for severe impact as the upper and lower structures met: he believes the impact may have been powerful enough to have been recorded by seismometers. [9] In his view, collapse initiation of the lower structure required “one powerful jolt.”[10] Of course, if there was a powerful jolt to the lower structure there must also have been a powerful jolt to the upper falling structure, in accord with Newton's Third Law.

In order to keep a sense of reality as we discuss NIST's theory it may be useful to label the three interacting parts of the North Tower, as they are pictured by NIST, as RB-12+, DS-6 and RB-92. Where RB stands for rigid block, DS stands for damaged structure, and the numbers following the letters refer to the number of stories in each structure. The upper block comprised the 12 stories of 99-110 as well as the roof structure with antenna and hat truss; the intermediate area was damaged by plane impact and fire and was six stories high (93-98 inclusive); and the lower block was rigid and comprised, in addition to subterranean levels, the first 92 stories of the building.

These designations actually underestimate the contrast between RB-12+ and RB-92, because the latter was not only largely undamaged by fire but was more massive per story. It was also stronger: the Tower's columns tapered as they ascended. [11] Yet the fall of RB-12+, we are supposed to believe, put a catastrophic end to DS-6 and RB-92.

What NIST essentially says, agreeing with Bazant, is that the lighter and weaker part initially fell with a powerful jolt onto the heavier and stronger part, which could not withstand its momentum, and that this caused a progressive collapse to initiate smashing the lower block to bits all the way to the ground.

The NIST Final Report does not tell us what happened to RB-12+ after its impact with the two structures beneath it. Did it fall through them all the way to the ground (that is, to the rubble heap on the ground), maintaining considerable mass and rigidity the whole time--as Bazant argued in 2001 and has continued to argue? [12]

On this the NIST authors are silent.

NIST also does not tell us how far RB-12+ fell before its impact with intact structure. Did it fall one story (roughly 12 feet), or several stories? We are left in the dark. Once again Bazant comes to the rescue. It fell “at least one story,” he says. [13]

To his credit, Bazant is willing to state the essential elements of the hypothesis. If this hypothesis is to hold any water at all there must be substantial impact: RB-12+ has a lot of work to do, so it had better fall at least one story.

As we will show, for the purposes of the present refutation it does not matter whether RB-12+ fell one story, six stories, or somewhere in between.

### **The Necessary Jolt:**

As Bazant has said, when the top part fell and struck the stories beneath it, there had to be a powerful jolt. While a jolt entails acceleration of the impacted object it requires deceleration of the impacting object. Even a hammer hitting a nail decelerates, and if the hammer is striking a strong, rigid body fixed to the earth its deceleration will be abrupt and dramatic.

Although NIST does not explicitly speak, like Bazant, of a “jolt”, and may therefore be thought to evade this paper’s refutation, it is impossible for NIST to escape the implications of its own assertions. The NIST report speaks of a strong, rigid structure (the upper structure or rigid block) falling freely onto another strong, rigid structure (the intact part of the building below the damaged area): the jolt cannot be avoided. [14]

*This was a necessary jolt.* Without it the required work could not have been done.

### **Testing for Deceleration:**

If a jolt occurred there would have been high short-term deceleration of the upper block. Why not simply check for this deceleration? It is not difficult. We will:

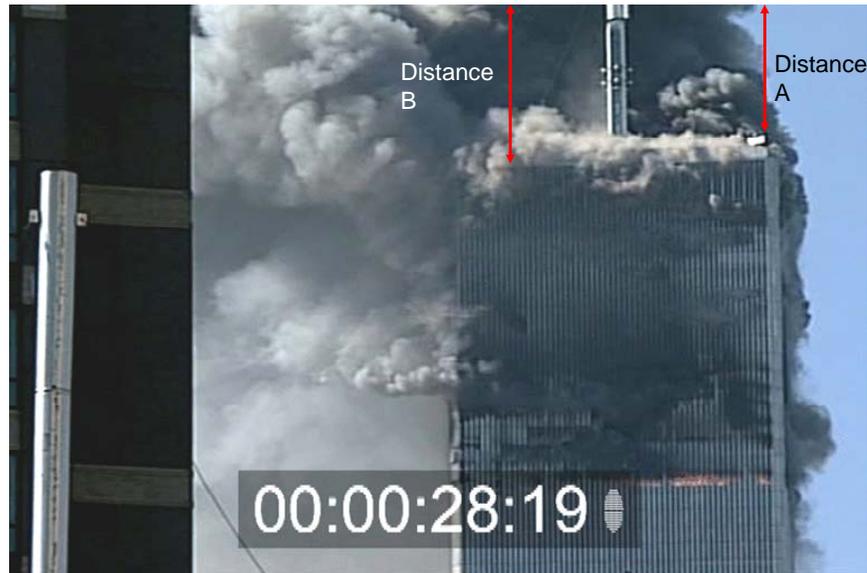
- examine a video clip of the North Tower’s collapse
- find a point on the upper block of the North Tower, the progress of which can be observed and measured in the early stages of the collapse
- plot the progress of this point on a graph
- check for evidence of deceleration

We have chosen a well known video clip of the collapse associated with French film maker, Etienne Sauret. [15] The Sauret clip has advantages over many others. It is a single, continuous sequence with no changes in camera angle and no zooming in and out. There is a very slight shift in the camera position relative to distant objects caused by a trembling of the camera several seconds prior to the collapse, but this is irrelevant to us since all our measurements are taken after the shift. The camera is very steady throughout the time we are making our measurements, as we can confirm by measuring the position of the picture frame relative to stationary objects. In addition, the image of the north face of the North Tower is exceptionally clear in these images.

Here is how we proceed: [16]

1. We save the Sauret footage to our hard drive.
2. We break the 1 minute, 56.53 second clip into 3497 equal segments or “frames.” Each frame is approximately 0.033 seconds in length (33 thousandths of a second).
3. We find two points associated with the roof of the upper block of the North Tower whose progress we can measure. Two points are necessary since neither one is consistently visible but one of the two is always visible. The point whose fall we shall use in our computations is at the tip of a white device on the roof. (The distance between this point and the upper frame is called Distance A in Figure 1 below.) The other point is located at the interface of the upper white section of the roof and the lower dark section. (The distance between this point and the upper frame is called Distance B in Figure 1.) The difference between Distance B and Distance A is approximately 28 pixels. Where the white device on the upper right-hand corner of the roof is obscured by smoke, measurements of the roof interface have been taken and the position of the device has been obtained by subtracting 28.
4. We choose a set of frames that stretches from Frame 929, before the discernible beginning of the roof’s fall, to the last frame in which our point can be recognized before it disappears into the dust cloud, Frame 1024.
5. We measure the number of pixels separating the white device from the fixed upper edge of the video frame, computing the position of the device when necessary by measuring the position of the roof interface. We take one measurement at each five frames in the progress of the Tower’s collapse, ending up with 20 points.
6. Our measurement stretches from 30.93 seconds into the clip to 34.1 seconds into the clip, giving us a total interval of 3.17 seconds.
7. We find that during this interval the white device on the roof has fallen a distance represented by 130 pixels.
8. In order to get an approximation of the real distances at issue; we find a known vertical distance on the north face of the North Tower. (The Tower’s proportions have been distorted as it has been rendered into frame-by-frame format. See Appendix A for a description of our method of determining the known vertical distance and the ratio of pixels to feet.) We discover that in our frame-by-frame version of the Sauret video 1 pixel = 0.88 feet. We now know that the point on the roof has fallen approximately 114.4 feet. The figure is not precise--there are the effects of foreshortening to consider (the roof and device are higher than the camera and the upper block, as it moves downward, tilts away from us)--but the figures are close enough for our purpose because we are looking for changes in acceleration over time, not exact velocity values.

Figure 1: Sauret Video: Representative Frame with Key Points for Measuring the Roof's Fall



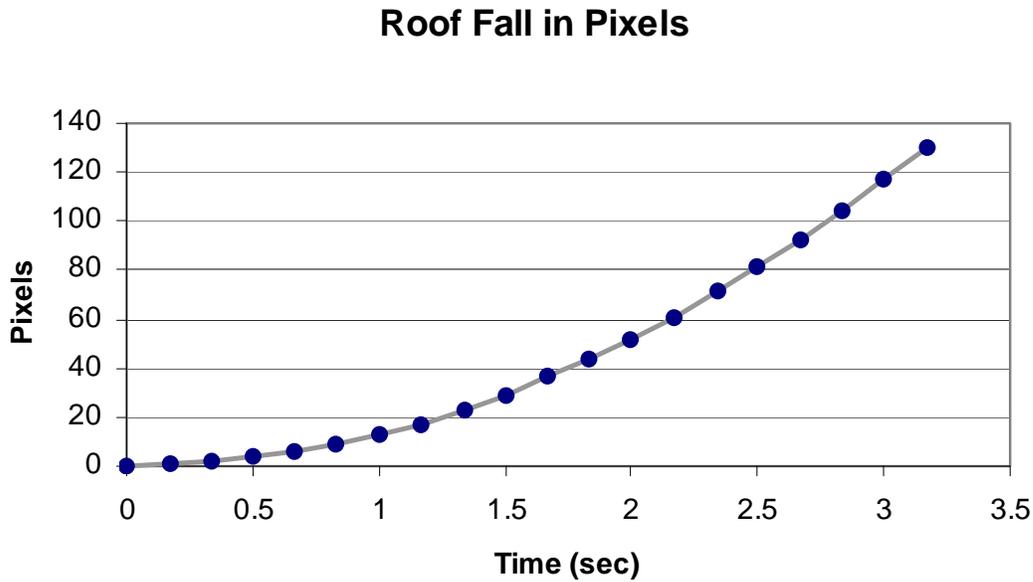
9. We know that

$$d = \frac{1}{2} \times g \times t^2$$

where  $d$  stands for distance,  $g$  stands for acceleration due to gravity, which is  $32.174 \text{ ft./s}^2$  at sea level, and  $t$  stands for time. Using this formula, we discover that a freely falling object would travel 161.6 feet in the time it took the roof to drop 114.4 feet.

10. We create two graphs. In the first the roof's descent is given in pixels. In the second the roof's fall is given in feet.

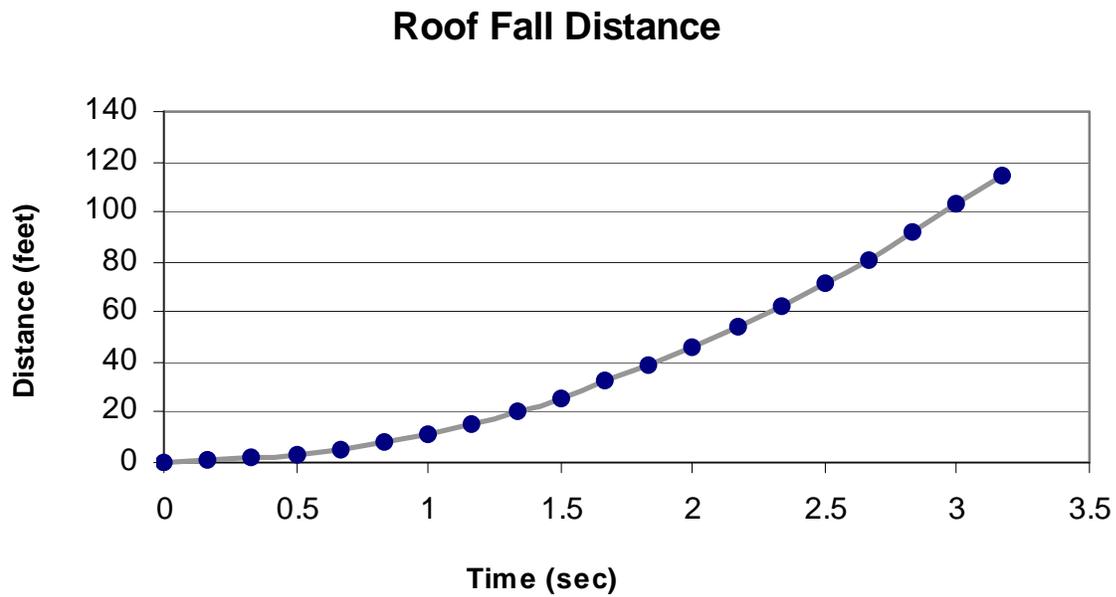
Figure 2: The Roof Fall: Pixels



Data:

Frame from clip start	Distance of device (pixels)	Distance of interface (pixels)	Distance of device (pixels) adjusted to 0	Distance of device (feet) adjusted to 0 (1 pixel = 0.88 ft.)	Time (seconds) from clip start	Time (seconds) adjusted to 0
929	109		0	0.00	30.93	0.0000
934	110		1	0.88	31.10	0.167
939	111		2	1.76	31.27	0.334
944	113		4	3.52	31.43	0.500
949	115		6	5.28	31.60	0.667
954	118		9	7.92	31.76	0.834
959	122		13	11.44	31.93	1.000
964	126		17	14.96	32.10	1.167
969	132		23	20.24	32.27	1.334
974	138		29	25.52	32.43	1.500
979	146		37	32.56	32.60	1.667
984	(153)	181	44	38.72	32.76	1.834
989	(161)	189	52	45.76	32.93	2.000
994	(170)	198	61	53.68	33.10	2.167
999	(180)	208	71	62.48	33.27	2.334
1004	(190)	218	81	71.28	33.43	2.500
1009	(201)	229	92	80.96	33.60	2.667
1014	(213)	241	104	91.52	33.76	2.834
1019	(226)	254	117	102.96	33.93	3.000
1024	(239)	267	130	114.40	34.10	3.167

Figure 3: The Roof Fall: Distance



Data:

Time (sec.)	Roof Fall Distance (ft.)
0.000	0.00
0.167	0.88
0.334	1.76
0.500	3.52
0.667	5.28
0.834	7.92
1.000	11.44
1.167	14.96
1.334	20.24
1.500	25.52
1.667	32.56
1.834	38.72
2.000	45.76
2.167	53.68
2.334	62.48
2.500	71.28
2.667	80.96
2.834	91.52
3.000	102.96
3.167	114.40

Knowing the distance the roof fell, in equal time intervals, from our measurements, we can now determine its actual velocity, at each measured point through its fall, using symmetric differencing. The equation is

$$V_n = (D_{n+1} - D_{n-1}) / (T_{n+1} - T_{n-1})$$

where V = velocity  
D = distance  
T = time  
n = point in question

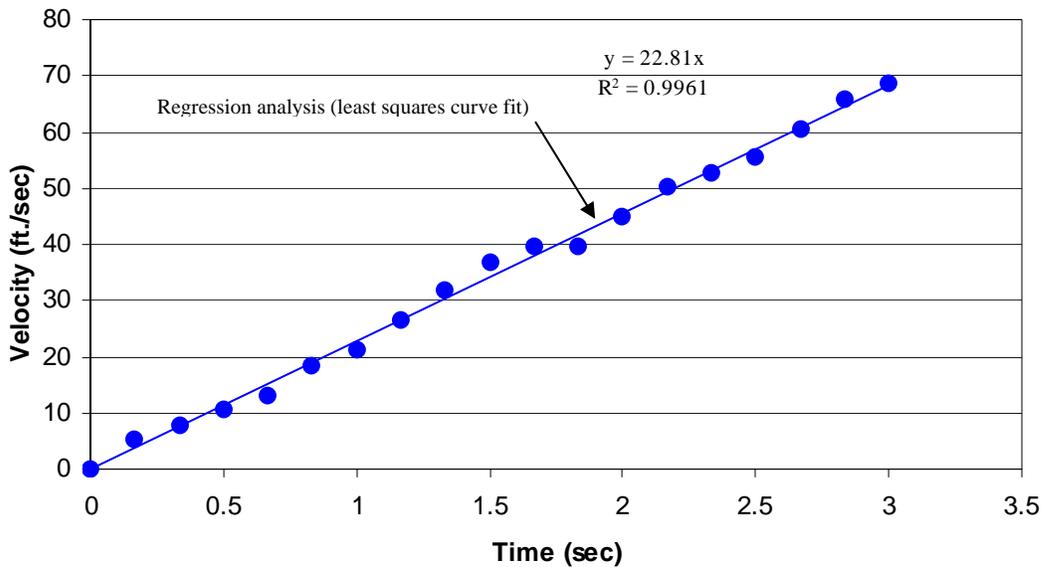
Data:

Time (sec)	Roof Fall Distance (ft.)	Velocity (ft./sec)
0.0000	0.00	0.00
0.1667	0.88	5.28
0.3334	1.76	7.92
0.5000	3.52	10.56
0.6667	5.28	13.20
0.8334	7.92	18.48
1.0000	11.44	21.12
1.1667	14.96	26.39
1.3334	20.24	31.68
1.5000	25.52	36.96
1.6667	32.56	39.59
1.8334	38.72	39.60
2.0000	45.76	44.88
2.1667	53.68	50.15
2.3334	62.48	52.81
2.5000	71.28	55.45
2.6667	80.96	60.71
2.8334	91.52	66.01
3.0000	102.96	68.65

It may be noticed that the last point measured at 3.167 seconds is left off of the data table above and the velocity graph below. The reason for this is that each point the velocity is found for needs to have a point ahead of it as well as behind it, so this method cannot calculate the velocity for the last point measured. As it is known that the measurements were taken every five frames with a 30 frame per second video, the actual time can be resolved fairly precisely. The use of four places for time increments, of 0.1667 seconds between measurements, in the velocity calculation above, is done for accuracy.

Below is a graph of the actual velocity of the roof at each measurement point over the same time frame in which the distance was measured.

Figure 4: The Roof Velocity



The velocity of the roof increases in a relatively linear way and is 68.65 ft./s after 3.00 seconds, which is about 71% of the free fall velocity of 96.52 ft./s for this fall time. At the actual measured distances and calculated velocities, the initial fall through one story would have taken place in approximately 1.0 second.

If the upper block, RB-12+, were rigid, as Bazant and NIST claim, the powerful jolt, required by Bazant to generate an impulsive load and explain the collapses of the Twin Towers, would show itself as an abrupt negative deviation in the otherwise positively sloped and virtually linear velocity graph.

For readers unfamiliar with the concept of an impulsive load, the impulse-momentum change equation is shown below and essentially shows that the change in momentum with respect to time provides the force involved in a collision.

$$Force = \frac{m_i v_i - m_f v_f}{\Delta t} = m \frac{\Delta v}{\Delta t} = ma$$

As stated earlier, it is only the velocity that changes with respect to the duration of the impulse, as the mass of an object is constant at all times everywhere in the universe. A change in velocity with respect to time is defined as either an acceleration or deceleration, depending on whether it is positive or negative. This acceleration or deceleration is then multiplied by the mass of the impacting object and provides the force involved in the collision, so the impulse equation ultimately reduces to the well known relation  $F = ma$ .

It is useful to refer to accelerations and decelerations in terms of the acceleration due to gravity, which is defined as 1g. The static weight of any item on earth is measured as the force due to the mass of the item multiplied by the acceleration of earth's gravitational pull or 1g. An acceleration or deceleration of 1g is equal to 32.174 ft./s<sup>2</sup>, so if the deceleration of an impacting object during a collision is greater than this then the weight or force applied by the impacting object is amplified. To find the number of g's involved one merely needs to divide the actual deceleration by 32.174 ft./s<sup>2</sup>.

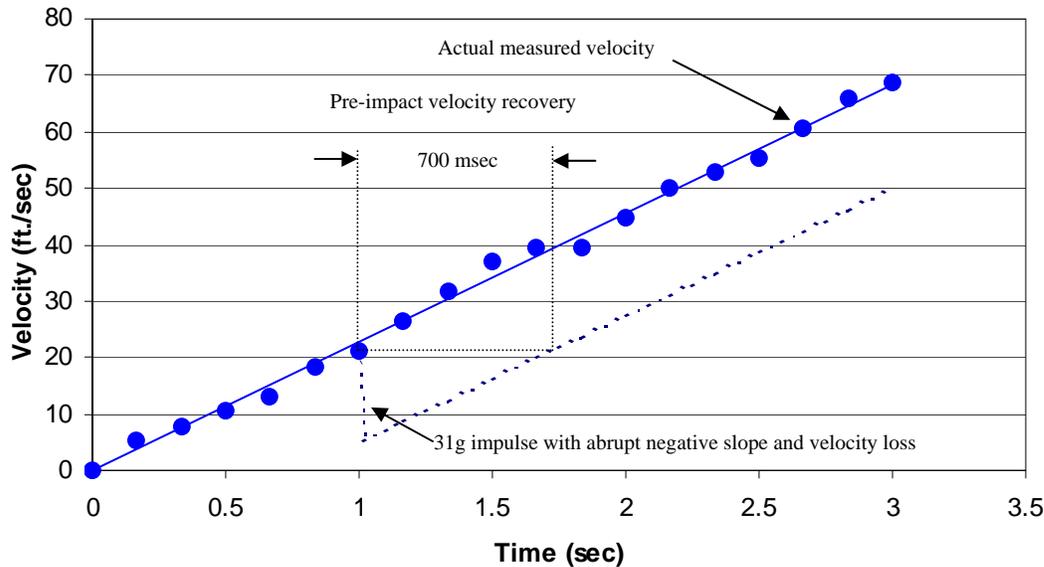
Bazant claims that a minimum force amplification of 31g, or 31 times the static weight of the upper stories, could have occurred in a collision between the upper and lower blocks of the Twin Towers after a fall of one story. [17] With the 98<sup>th</sup> story columns completely collapsing, a distance between floor slabs of approximately 11.44 feet, and the actual measured velocity of 22.81 ft./s of the upper block at this point, the first collision would have occurred approximately one second into the fall. Regardless of the actual amplification, any impulse at the impact zone between the 98<sup>th</sup> and 99<sup>th</sup> story floor slabs capable of causing collapse continuation would have had to cause the columns on at least the first stories on either side of the impact to deform elastically, and plastically, and then to buckle. The deformations and buckling of the columns of the impacting stories, on both the lower and upper blocks, would cause a kinetic energy drain, which would reduce the velocity of the rigidly attached falling mass above them. Using energy methods we have calculated what effect these energy drains would have on the velocity of the upper block. Since the upper block would pick up the mass of the 98<sup>th</sup> floor in the impact there would also be a conservation of momentum component to the velocity reduction. From Appendices D and E we find the reduced velocity ( $V_{\text{reduced}}$ ) of the upper block, after impact, considering the three energy drains and conservation of momentum, and it is

$$\begin{aligned} V_{\text{reduced}} &= 22.81 \text{ ft./s} - (15.63 \text{ ft./s} + 1.75 \text{ ft./s}) \\ &= 22.81 \text{ ft./s} - 17.38 \text{ ft./s} \\ &= 5.43 \text{ ft./s} \end{aligned}$$

Since the roof was part of the rigid upper block it would have displayed this momentary abrupt change in its velocity, from 22.81 ft./s to 5.43 ft./s, if the collapse were a natural occurrence. It should also be noted that the energy losses and conservation of momentum we have calculated and used here, to determine the velocity loss, are a minimum. We do not consider energy losses due to vibration of the building, heat, and sound, during the initiating impulse, all of which would have required energy from the impulse to produce and thus have an additional effect on velocity loss. The intent here is only to show that, even with a quantifiable minimum energy loss and conservation of momentum, the velocity loss would be quite dramatic, and should have been readily observed if an impulse capable of causing collapse had indeed occurred.

The graph below shows what the upper block velocity change would look like if a 31g impulse had occurred one story into the fall, with its velocity at least momentarily reduced in a significant way after impact.

Figure 5: Roof Velocity Curve with a hypothetical 31g deceleration



The fact that a 31g impulse requires a deceleration of  $997.4 \text{ ft./s}^2$  is unassailable, and it does not matter whether the collision is elastic or inelastic. With a velocity reduction of  $17.38 \text{ ft./s}$  and a  $997.4 \text{ ft./s}^2$  deceleration, the duration of this impulse would have been 17 milliseconds. This rapid deceleration associated with the 31g impulse would necessarily show itself as an abrupt negative slope change in the velocity curve.

We have shown the curve starting upward again after the impact, on the generous assumption that the impacting object (the upper block) is now free to accelerate. We have also only charted what the effect on the velocity would have been for an initiating impulse between the first two floors to collide.

The measurements of the roof's actual fall do not show any abrupt negative change in velocity, so it appears that there was no impulse and thus no amplified load. It seems that Dr. Bazant was simply theorizing that there had to be one to make sense of the collapse in a natural way. It is also important to note here that Dr. Bazant was off by a factor of ten in his calculation of the stiffness of the columns, with his  $71 \text{ GN/m}$  estimate. [8] The actual stiffness, calculated here using the actual column cross sections, is approximately  $7.1 \text{ GN/m}$ . (see Appendices B and C) [19][20] This error caused Dr. Bazant to significantly overestimate the potential amplifying effect of the impulse or jolt, which he claims occurred after a one story fall of the upper block.

In an effort to refute the argument put forth in this paper, some may claim that plastic deformation of the lower stories of the upper block could have created a crush wave below the upper block and kept the roof from experiencing a discernable impulse. If that were true then the impulse durations would have increased dramatically, absorbing the energy over a longer period of time and eliminating any significant amplification of the upper block's weight. But without the amplification of the upper block's weight why would the lower block have collapsed?

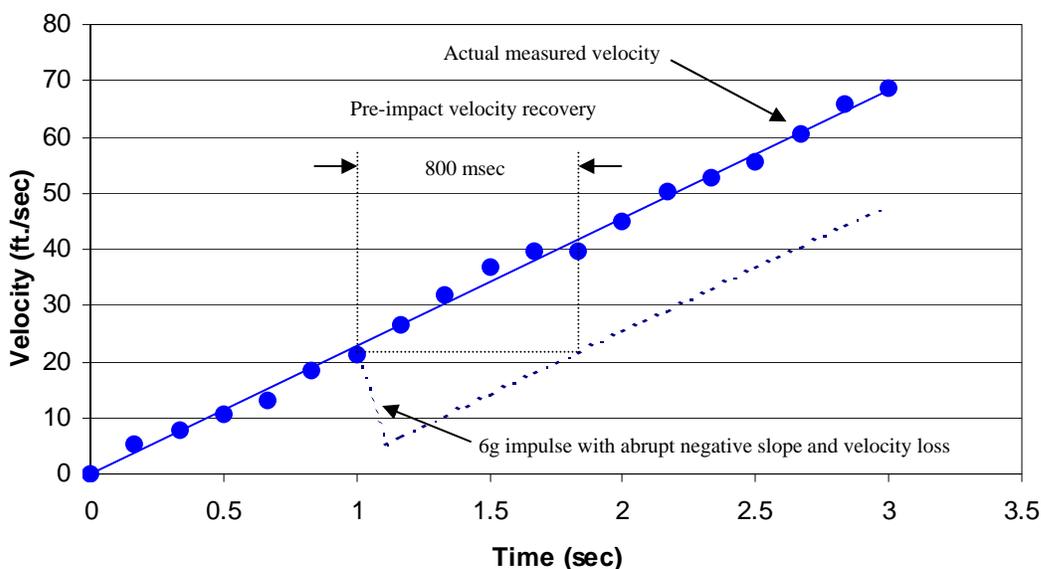
There are those who might argue that the tilt of the upper block to the south could have kept an

impulse from being discernable—that there may have been impulses on the south face or further inside the Tower, in the central core, that were not visible on the north face. Impulses at these locations could not have caused the collapse of the north face of the Tower and its corner columns in the observed vertical manner. The corner columns of the east and west faces, in conjunction with the columns of the north face, formed a structural channel (a stiff structural element with support in two orthogonal directions) and, barring planned demolition, would have collapsed as observed only if they were struck impulsively, in a vertical manner by the upper block.

In reality, the upper block could not have tolerated the potential 31g impulse theorized by Dr. Bazant. To get this overload he claims was possible, all of the mass of the upper block would have had to participate, and if it did so it would have come apart completely.

Perhaps the impulse was of a lower value but still high enough to cause an overload of the lower structure and bring about global collapse? Consider a velocity graph with a 6g deceleration, very likely the minimum load amplification necessary to overcome the reserve capacity of the perimeter columns, which had a minimum factor of safety of 5.00 to 1.

Figure 6: Roof Velocity Curve with a hypothetical 6g deceleration



A 6g impulse requires a deceleration of  $193 \text{ ft./s}^2$ . With a velocity reduction of  $17.38 \text{ ft./s}$  and a  $193 \text{ ft./s}^2$  deceleration, the duration of this impulse would have been 90 milliseconds. As the graph shows, there would still be a quite obvious abrupt negative slope change, which is not seen in the velocity curve determined from the measured data.

The measurements were taken every five frames, or 167 milliseconds apart. The recovery to the pre-impact velocity is shown to occur in the dashed graphs in the approximate times of 700 milliseconds for the 31g case and 800 milliseconds for the 6g case. In both cases there are four data points taken well within this recovery window, so it is apparent that a negative change in the velocity of the roof would have been captured if an impulse had indeed occurred.

## **Findings:**

As the figures and graphs above clearly show, any impulsive load would have required a high deceleration, which would have shown itself very prominently in the velocity curve derived from the measured data. The fact that no such negative change exists in the roof's actual velocity curve reveals that no major interruption or significant abrupt deceleration, and therefore no amplified load, could have occurred during the fall of the upper block. How can this be? If RB-12+ fell with a jolt on the rest of the building after a 12 foot drop (one story), the deceleration, as shown above, would have revealed itself clearly, and if RB-12+ fell more than one story, the deceleration would have been even more dramatic. If RB-12+ fell 72 feet—all the way through the six damaged stories—we would see powerful evidence of a jolt during the measured 114.4 foot fall of the roof. It would be dramatic precisely because the velocity and therefore the momentum would be high, and any change more discernable. But there is no evidence of major impact and deceleration either early or late.

In the main, these findings confirm the earlier research of Dr. Frank Legge. [18] In 2006 Legge, using a different video clip and measurement technique, carried out detailed measurements of the fall of the roof of the North Tower and calculated its acceleration rate. Although his purposes were different from ours, he discovered similarly smooth curves. There is no more trace of deceleration in his graphs than in ours.

What happened to RB-12+ during its fall? It would appear, based on the Sauret video and other video recordings of the event, that a substantial portion of the bottom of RB-12+, along with DS-6, was violently destroyed amidst clouds of ejected matter at the same time the top portion of RB-12+, containing the rooftop, was falling. Since the clouds of matter in the videos obscure many details of the event, it is easy to see why someone might try to make the case that the fall of the upper portion of the rigid block was accompanied by a fall of its lower portion. But we do not see a fall of its lower portion: we simply see violent destruction in the vicinity of the lower portion and fall of the upper portion.

To repeat: if RB-12+ had fallen as a rigid block, there would be impact, and the impact would have caused abrupt interference with the fall of its upper part, including the roof. No such interruption has occurred, and therefore no such impact has taken place. Evidently, the violent destruction that occurred--presumably through planted explosives or other means of demolition--effectively destroyed the structural integrity of the lower part of the upper block as well as DS-6, permitting the upper block to fall at speed while meeting minimal resistance and experiencing neither major impact nor abrupt deceleration.

## **Conclusions:**

*We have tracked the fall of the roof of the North Tower through 114.4 feet, (approximately 9 stories) and we have found that it did not suffer severe and sudden impact or abrupt deceleration. There was no jolt. Thus there could not have been any amplified load. In the absence of an amplified load there is no mechanism to explain the collapse of the lower portion of the building, which was undamaged by fire. The collapse hypothesis of Bazant and the authors of the NIST report has not withstood scrutiny.*

## NOTES

Thanks are offered to members of the discussion forum of Scholars for 9/11 Truth & Justice, especially to Alfons, who initiated the discussion and provided a number of interesting ideas. Thanks are also due to Zoran Bilanovic for a critical reading of the paper and to Paul Bouvet for early software advice. Crucial software assistance was obtained from Joe Terrien, who gave freely of his time and expertise. We are enormously grateful to Civil Engineering Professor Robert Korol for help with the calculations in the appendices. All measurements, calculations, and conclusions are the sole responsibility of the authors.

1. NIST NCSTAR 1. Federal Building and Fire Safety Investigation of the World Trade Center Disaster. Final Report on the Collapse of the World Trade Center Towers. National Institute of Standards and Technology, U.S. Department of Commerce, September 2005.

<http://wtc.nist.gov/NISTNCSTAR1CollapseofTowers.pdf>

2. There is some ambiguity in the NIST study on which stories are included in the upper rigid block, but the analysis given in this paper appears to represent NIST's best estimate. See, e.g., NIST NCSTAR 1, p. 150-151.

3. NIST NCSTAR 1, p. 151.

4. NIST NCSTAR 1, p. 151.

5. NIST NCSTAR 1, p. xxxviii.

6. NIST NCSTAR 1, p.151.

7. NIST NCSTAR 1, 145 (“the falling building section”); 146 (“the falling building mass,” “the falling mass”). See also the companion volume: NIST NCSTAR 1-6. Federal Building and Fire Safety Investigation of the World Trade Center Disaster. Structural Fire Response and Probable Collapse Sequence of the World Trade Center Towers. National Institute of Standards and Technology, U.S. Department of Commerce, September 2005, p. liv (“the building section began to fall downward”) and p. 156 (“the building section began to fall vertically”).

<http://wtc.nist.gov/oct05NCSTAR1-6index.htm>

8. Bazant, Zdenek and Yong Zhou, “Why Did the World Trade Center Collapse?—Simple Analysis”. *Journal of Engineering Mechanics*, vol. 128, no. 1 (Jan. 2002), p. 2-6 See also the Addendum to this article in the same journal, vol. 128, no. 3 (March 2002), p. 369-370.

<http://www.civil.northwestern.edu/people/bazant/PDFs/Papers/405.pdf>

Note: when we refer in the article to Bazant, we include his co-authors.

For NIST's reference to the Bazant paper, see NIST NCSTAR 1-6, p. 323.

9. Bazant, Zdenek and Jie-Liang Le, Frank Greening, David Benson, “Collapse of the World Trade Center Towers: What Did and Did Not Cause It?” Structural Engineering Report No. 07-05/C605c. Department of Civil and Environmental Engineering, Northwestern University. May 27, 2007. Revised December 15, 2007. p. 11.

<http://www.civil.northwestern.edu/people/bazant/PDFs/Papers/00%20WTC%20Collapse%20-%20What%20did%20%26%20Did%20Not%20Cause%20It%20-%20Revised%206-22-07.pdf>

Jim Hoffman challenged Bazant’s claim some time ago in his article “Seismic Records of the Twin Towers’ Destruction: Clarifying the Relationship Between Seismic Evidence and Controlled Demolition Theories.” Version 0.9, Oct. 31, 2006.

[http://911research.wtc7.net/essays/demolition/seismic.html#evidence\\_of](http://911research.wtc7.net/essays/demolition/seismic.html#evidence_of)

10. Addendum, Bazant and Zhou, 2002, p. 369.

11. See Gregory Urich, “Analysis of the Mass and Potential Energy of World Trade Center Tower 1.” Journal of 9/11 Studies, vol. 18 (Dec. 2007).

<http://www.journalof911studies.com/>

See also NIST NCSTAR 1-1. Federal Building and Fire Safety Investigation of the World Trade Center Disaster. Design, Construction, and Maintenance of Structural and Life Safety Systems. National Institute of Standards and Technology, U.S. Department of Commerce, September 2005.

<http://wtc.nist.gov/oct05NCSTAR1-1index.htm>

12. Bazant and Zhou, 2002; Bazant et al, 2007.

13. Bazant et al, 2007, p. 1.

14. The following four points commit NIST to impact and jolt:

(a) NIST speaks of the core of the building as consisting of three sections, which correspond closely to the sections we have spoken of when discussing the building as a whole:

“At this point, the core of WTC 1 could be imagined to be in three sections. There was a bottom section below the impact floors that could be thought of as a strong, rigid box, structurally undamaged and at almost normal temperatures. There was a top section above the impact and fire floors that was also a heavy, rigid box. In the middle was the third section, partially damaged by the aircraft and weakened by heat from the fires.” (NIST NCSTAR 1, p. 79)

(b) The section of the building above the damage zone NIST calls a “rigid block.” This rigid block first manifests its independent movement when it tilts to the south. (“The section of the building above the impact zone (near the 98<sup>th</sup> floor), acting as a rigid block, tilted...” NIST NCSTAR 1, p. 201.) NIST also refers to this rigid block with terms such as “upper section,” “building section above the impact zone,” “building mass,” “upper building section” and “structural block.” See NIST NCSTAR 1, pp. 83, 195, 196, 201

(c) NIST acknowledges that this rigid block then *falls*. NIST says that “the building section began to fall downward,” “the building section began to fall vertically.” Indeed, we are told that this falling rigid block goes through all or part of the damaged area “essentially in free fall.” (“Since the stories below the level of collapse initiation provided little resistance to the tremendous energy released by the falling building mass, the building section above came down essentially in free fall, as seen in videos.”) See NIST NCSTAR 1-6, pp. 416, 238; NIST NCSTAR 1, p. 196.

(d) After falling through all or part of the damaged area of the tower, the rigid block or falling building mass encounters “intact structure.” (“The potential energy released by the downward movement of the large building mass far exceeded the capacity of the intact structure below to absorb that through energy of deformation.”) See NIST NCSTAR 1, p. 196. This “intact structure,” has, of course, already been referred to as including the core of the building, described as “a strong, rigid box, structurally undamaged and at almost normal temperatures.”

15. A version of the Sauret video clip can be found at:

<http://www.youtube.com:80/watch?v=xGAofwkAOlo>

For our purposes we have used the footage from Etienne Sauret’s film, “WTC: the first 24 hours.”

16. Readers wanting to get a rough approximation of the measurements in this paper without expense may acquire from the internet the software, Vdownloader:

<http://www.softpedia.com/progDownload/VDownloader-Download-51327.html>

Once the on-line version of the Sauret video clip (see note 15) is downloaded it can be broken into 0.033 second frames using VirtualDub:

<http://www.virtualdub.org/>

A pixel measurement device (several are available free or for a minimal charge on the Internet) can be used for measurements.

For our paper we found we were able to get more accurate measurements by ripping the Sauret video (from the DVD) using DVD Decrypter. Then the raw video files were converted to mpeg2 using Xilisoft Video Converter 3. The converted files were then imported into Adobe Premiere Pro CS3. The timestamp was added and the entire segment was exported as a still frame sequence in .gif format.

For pixel measurements, we used Screen Calipers:

<http://iconico.com/caliper/>

17. Bazant and Zhou, 2002, p. 3.

18. Frank Legge, “9/11-Evidence for Controlled Demolition: a Short List of Observations.” Journal of 9/11 Studies, vol.1 (June 2006).

[http://www.journalof911studies.com/articles/Journal\\_2\\_Evidence\\_for\\_demolition\\_20.pdf](http://www.journalof911studies.com/articles/Journal_2_Evidence_for_demolition_20.pdf)

19. The cross sectional areas of the central core columns on each story were released by NIST in 2007 and are publicly available. This information can be found at <http://wtcmodel.wikidot.com/nist-core-column-data>

20. The exterior column cross sectional area for each story was determined using the WTC1 mass analysis cited in reference [11], which gives the total mass of the columns on each story. Knowing the length of the columns and the density of steel, the area could be determined.

21. See Section 2.6 on pages 5 through 7 of the below link for an explanation of column cross section classifications for resistance to local buckling.  
<http://www.nottingham.ac.uk/civeng/H23S07/Design%20of%20SHS.pdf>

\* This paper has been revised to use symmetric differencing to calculate instantaneous velocity. The initial method used the equations of motion to calculate velocity, which are only valid with constant acceleration, causing smoothing of the data and inflation of the pre-impact velocity. Since the energy requirements do not change, the actual lower pre-impact velocity results in a larger percentage of kinetic energy drained at impact with a correspondingly more dramatic change in velocity.

An arithmetic error in the velocity reduction calculation on page 26 was also corrected.

## AUTHORS

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## APPENDIX A

### DETERMINING THE PIXEL-FOOT RATIO FOR THE SAURET VIDEO

1. In order to correct any possible vertical distortion of the image of the North Tower that might affect our measurements (such distortions are common), we decided to find a vertical distance on the north face of the Tower that can be measured accurately in pixels. We took a measurement from a horizontal line of damage caused by the plane to a line on the roof of the NT, where the upper white part of the roof meets a darker, lower part of the building.

2. We then chose five excellent still photos of the North Tower. The perspective from which they were taken seemed unlikely to create severe foreshortening effects. These photographs are from the NIST report (NIST NCSTAR 1.5A, Chapter 8), and are grouped conveniently on the forensic website “WTC Demolition Analysis” found at:

[http://www.sharpprintinginc.com/911/index.php?module=photoalbum&PHPWS\\_Album\\_id=20&PHPWS\\_Photo\\_op=view&PHPWS\\_Photo\\_id=909](http://www.sharpprintinginc.com/911/index.php?module=photoalbum&PHPWS_Album_id=20&PHPWS_Photo_op=view&PHPWS_Photo_id=909)

The photographs were taken at different times and by several different photographers, and they are reproduced below with added red arrows showing the two distances measured.

Our aim was to measure, in pixels, the horizontal distance  $x$  and then the vertical distance  $y$  so that we could work out the ratio of  $x$  to  $y$ . If consistency could be found, we could be confident that we had the correct ratio. Then, knowing the value of  $x$  (the width of the tower) in feet, we could determine the value, in feet, of  $y$ .

3. Here are the measurements made for the original five photos, marked A, B, C, D, and E. (Note that the measurements will be different on the photos as reproduced below, but the proportions will remain constant.)

Photo	$x$	$y$	$x:y$ ratio
A	231 pixels	211 pixels	1: .91
B	373 pixels	340 pixels	1: .91
C	379 pixels	354 pixels	1: .93
D	373 pixels	343 pixels	1: .92
E	327 pixels	302 pixels	1: .92

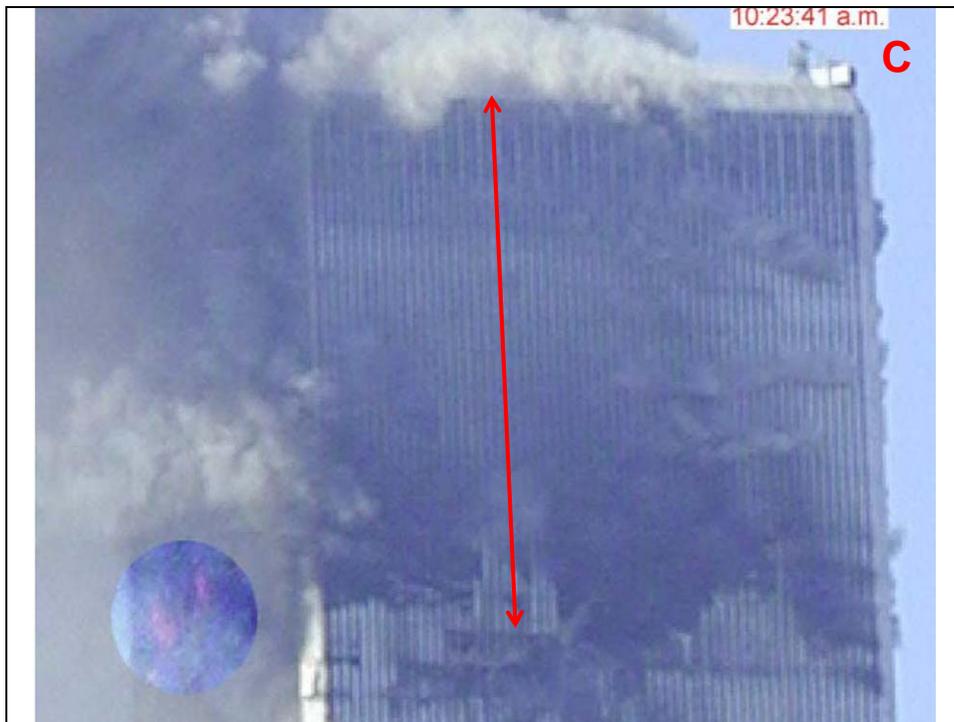
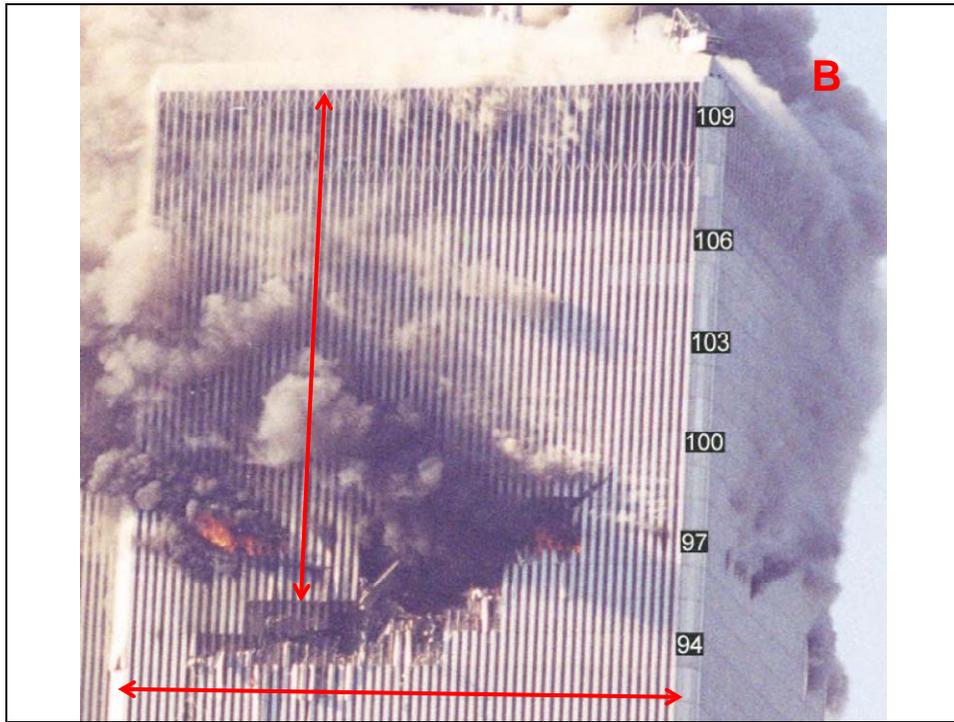
4. There is little variation in the figures found for the ratio of  $x: y$ . The average is 1:92, which corresponds to the ratio in what is arguably the photograph with the least apparent distortion from foreshortening, photo D,

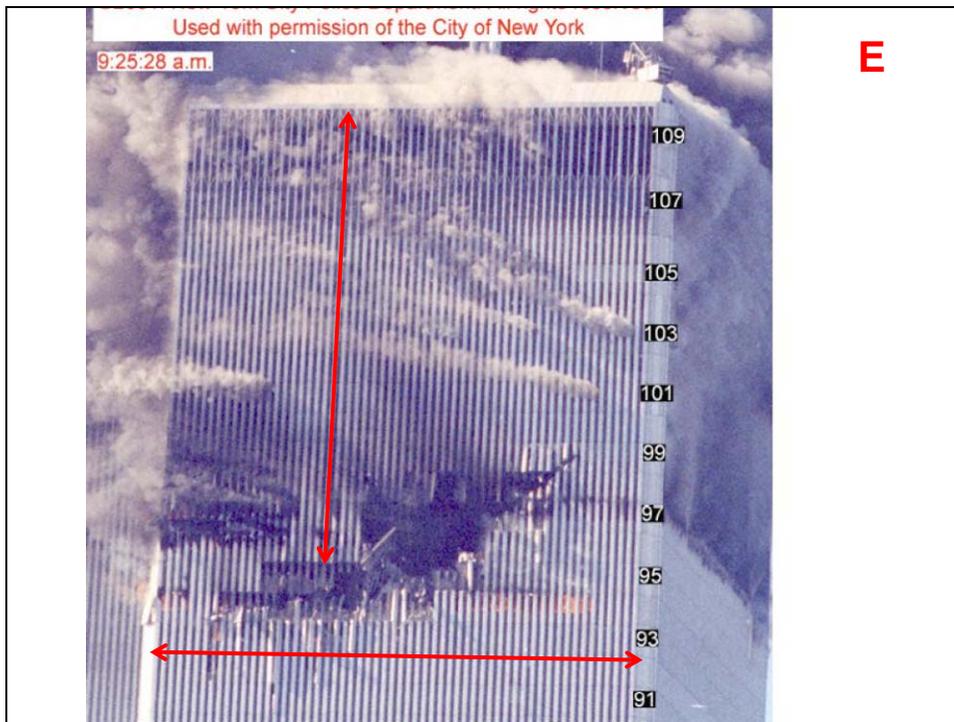
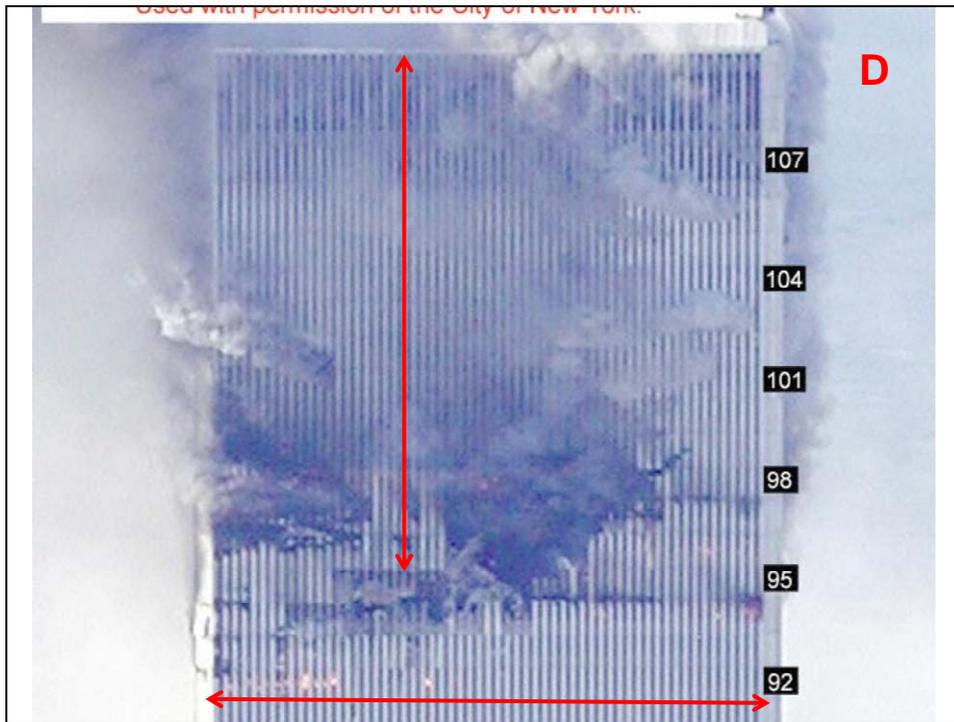
5. Various figures, from 207 to 210 feet, have been suggested for the external width of the Towers. We chose 210 feet as our best estimate. The figure is from NIST NCSTAR 1, p. 5. See also Gregory Urich, “Analysis of the Mass and Potential Energy of World Trade Center Tower 1” (*Journal of 9/11 Studies*), p. 8. Bear in mind that the perimeter columns were covered in insulation and aluminum cladding, which added to their external dimensions.

6. This means that the value of the vertical distance measured ( $y$ ) is  $210 \times .92 = 193.2$  feet.

7. Measuring  $y$  in our frame-by-frame version of the Sauret video we found it to be 220 pixels. The ratio of pixels to feet for vertical measurements in this version of the Sauret video is: 1 pixel = 0.88 feet.







## APPENDIX B

### CORE AND PERIMETER COLUMN CROSS SECTIONS ON THE 97<sup>TH</sup> STORY

#### CORE COLUMNS

97 <sup>TH</sup> story		Yield	Flange	Flange	Web	Web	Cross sectional
Column No.	Designation	strength (ksi)	width (in.)	thickness (in.)	height (in.)	thickness (in.)	area (in. <sup>2</sup> )
501	14WF426F42	42	16.695	3.033	12.624	1.875	124.942
502	14WF264F42	42	16.025	1.938	12.624	1.205	77.325
503	14WF264F42	42	16.025	1.938	12.624	1.205	77.325
504	14WF246F36	36	16.945	1.813	12.624	1.125	75.645
505	14WF219F36	36	15.825	1.623	12.624	1.005	64.055
506	14WF287F42	42	16.13	2.093	12.624	1.31	84.058
507	14WF264F42	42	16.025	1.938	12.624	1.205	77.325
508	14WF426F42	42	16.695	3.033	12.624	1.875	124.942
601	12WF106F42	42	12.23	0.986	10.908	0.62	30.881
602	14WF150F36	36	15.515	1.128	12.624	0.695	43.776
603	14WF158F36	36	15.55	1.188	12.624	0.73	46.162
604	12WF106F36	36	12.23	0.986	10.908	0.62	30.881
605	12WF120F36	36	12.32	1.108	10.908	0.71	35.046
606	14WF150F36	36	15.515	1.128	12.624	0.695	43.776
607	14WF142F36	36	15.5	1.063	12.624	0.68	41.537
608	12WF106F36	36	12.23	0.986	10.908	0.62	30.881
701	12WF161F42	42	12.515	1.486	10.908	0.905	47.066
702	14WF176F36	36	15.64	1.313	12.624	0.82	51.422
703	14WF103F36	36	14.575	0.813	12.624	0.495	29.948
704	14WF53F50	50	8.062	0.658	12.624	0.37	15.280
705	14WF43F45	45	8	0.528	12.624	0.308	12.336
706	14WF111F36	36	14.62	0.873	12.624	0.54	32.343
707	14WF167F36	36	15.6	1.248	12.624	0.78	48.784
708	12WF161F42	42	12.515	1.486	10.908	0.905	47.066
801	12WF161F42	42	12.515	1.486	10.908	0.905	47.066
802	14WF176F36	36	15.64	1.313	12.624	0.82	51.422
803	12WF133F42	42	12.365	1.238	10.908	0.756	38.862
804	12WF79F36	36	12.08	0.736	10.908	0.47	22.909
805	14WF111F42	42	14.62	0.873	12.624	0.54	32.343
806	14WF167F36	36	15.6	1.248	12.624	0.78	48.784
807	14WF167F42	42	15.6	1.248	12.624	0.78	48.784

901	12WF120F36	36	12.32	1.108	10.908	0.71	35.046
902	14WF150F36	36	15.515	1.128	12.624	0.695	43.776
903	14WF193F36	36	15.71	1.438	12.624	0.89	56.417
904	12WF92F36	36	12.155	0.856	10.908	0.645	27.845
905	12WF99F36	36	12.19	0.921	10.908	0.56	28.562
906	14WF142F36	36	15.5	1.063	12.624	0.68	41.537
907	14WF150F36	36	15.515	1.128	12.624	0.695	43.776
908	12WF120F36	36	12.32	1.108	10.908	0.71	35.046
1001	14WF426F42	42	16.695	3.033	12.624	1.875	124.942
1002	14WF264F42	42	16.025	1.938	12.624	1.205	77.325
1003	14WF342F36	36	16.365	2.468	12.624	1.545	100.282
1004	14WF219F36	36	15.825	1.623	12.624	1.005	64.055
1005	14WF202F36	36	15.75	1.503	12.624	0.93	59.085
1006	14WF314F42	42	16.235	2.283	12.624	1.415	91.992
1007	14WF287F36	36	16.13	2.093	12.624	1.31	84.058
1008	14WF426F42	42	16.695	3.033	12.624	1.875	124.942

Total area (in.<sup>2</sup>) = 2,621.657

### PERIMETER COLUMNS

The perimeter columns were uniform in cross section on a given floor. While their exact cross sections have not been made publicly available they are discernable due to their height, number, material density, and total weight per floor being known. The NIST NCSTAR 1-3D report states that “As the elevation in the building increased, the thickness of the plates in the columns decreased, but the plates were always at least 0.25 thick”.

The height of a floor of perimeter columns in WTC 1 can be calculated by dividing the building height of 1,368 feet by 110 stories to get a height of 12.44 feet or 149.24 inches per story.

The weight of the 236 perimeter columns at the 97<sup>th</sup> story was approximately 78.71 tons or 157,420 lbs..

Dividing the weight by the 0.283 lbs./in.<sup>3</sup> density of steel and the number of columns gives a volume for each column of 2,357 in.<sup>3</sup>.

Dividing this volume by the 149.24 inch height of each floor gives a cross sectional area for each column of 15.79 in.<sup>2</sup>.

With 236 columns this gives a total cross sectional area for the perimeter columns at the 97<sup>th</sup> story of 3,726 in.<sup>3</sup>.

As the perimeter columns can be approximated as 14 inch square columns, the wall thickness can be estimated. For the 97<sup>th</sup> story it would be approximately 0.289 inches. This comports well with the NIST statement that the plate thickness was never less than 0.25 inches thick, and since the 97<sup>th</sup> story was 13 floors down from the top of the building it appears reasonable.

## APPENDIX C

### CALCULATION OF THE AXIAL STIFFNESS OF THE COLUMNS FROM THE 97<sup>TH</sup> STORY DOWN TO GROUND LEVEL IN THE TOWERS

The axial stiffness of a structural column can be determined knowing the modulus of elasticity of its material, the cross sectional area, and the length of the column, with the equation  $K = AE/L$ .

The problem for determining this for the tower columns below the 97<sup>th</sup> story is that the cross sectional areas change with elevation. One way to estimate the cross section is to use a median, which we will do here using the cross sectional area of the columns at the 55<sup>th</sup> story since it is the midpoint in the tower above ground level.

The core column cross sectional area at the 55<sup>th</sup> story was 8,777 in.<sup>2</sup> and the perimeter column cross sectional area 10,784 in.<sup>2</sup> giving a total column cross sectional area of 19,561 in.<sup>2</sup> at the 55<sup>th</sup> floor.

Steel was used for all of the columns and the modulus of elasticity of steel is  $30 \times 10^6$  psi.

The length of the columns from the 97<sup>th</sup> story down to ground level was 149.24 inches per story multiplied by 97 stories, giving a length of 14,476 inches.

Using  $K = AE/L = (19,561 \text{ in.}^2)(30 \times 10^6 \text{ psi})/14,476 \text{ inches}$ , the stiffness is found to be 40,538,132 lb./in. or 7.1 GN/m.

While one could make the case that the stiffness used should have been that from the 97<sup>th</sup> story down to the foundation, and considering the six sub-levels, the stiffness in that case would be nearly the same. The median floor in that case would be the 52<sup>nd</sup> story and the columns on that floor were only slightly larger in cross section than those on the 55<sup>th</sup>, which would be offset in the calculation by the additional length of the six sub-level floors.

## APPENDIX D

### CALCULATION OF VELOCITY CHANGES DUE TO ENERGY DRAINS DURING THE COLLISION OF THE UPPER AND LOWER BLOCKS

It is assumed that there are 3 parts to the energy dissipation from the collision for a given story. These are:

- 1) Uniform elastic spring action compression in the core and perimeter columns.
  - 2) Compressive plastic yielding of core and perimeter columns in columns of the 97<sup>th</sup> and 99<sup>th</sup> stories.
  - 3) Plastic hinging action (buckling) of all columns, in the two stories.
- 1) Calculations show that an average spring constant for the tower columns is 40,500 kips/in or 7.1 GN/m, i.e. if the columns were of uniform cross section over the 110 stories of the building, and using values found at mid-height for the 55<sup>th</sup> story. If the cross sections were uniform the tops of the columns of the 97<sup>th</sup> story would axially compress elastically 19.84 in. However, the column sections are not uniform, since the cross sections get smaller with increasing height, as one would expect with decreasing load. Thus the 97 stories of columns can only be shown to compress elastically the amount consistent with the least cross sectional area, i.e. those of the 97<sup>th</sup> story. To calculate the maximum resistance offered by the core and perimeter columns in the 97<sup>th</sup> story we need to take into account the fact that some columns are very stocky while some have thin elements that will buckle locally before they yield. All 47 core columns plus 236 perimeter columns are categorized into classes 1, 2, 3 and 4 (with 4 being the thinnest-walled and 1 being the sturdiest), where class 4 columns do not reach yield before local buckling occurs. [21] Approximately half of the core columns were 36 ksi yield strength with the remaining half at 42 ksi or above, resulting in an average yield strength of approximately 40 ksi. 14 of the core columns are class 4 and we conservatively use 50% yield resistance before buckling for these columns. With the remaining 33 columns being given 40 ksi credit, we get a total core column load resistance of 94,900 kips. The 236 perimeter columns at the 97<sup>th</sup> story are considered class 4, but all have a yield strength of 65 ksi. Using the 1/2 factor and multiplying by the total area of perimeter columns we get 121,600 kips. The total sustainable load, before plastic deformation occurs, for the 97<sup>th</sup> story columns = 216,500 kips. As expected, the columns of the 55<sup>th</sup> story have a significantly larger overall cross section and their sustainable load, before plastic deformation occurs, is 821,600 kips. The elastic displacement of the tops of the 97<sup>th</sup> story columns can then be found using the ratio of (216.5/821.6) times 19.84 inches = 5.22 inches. Using the equation  $E = \frac{1}{2}Kx^2$ , the elastic energy absorbed by axial deformation of the columns can be calculated using the figures above as  $\frac{1}{2} (40,500 \text{ kips/in.})(5.22 \text{ in.})^2 = 552,000 \text{ in-k}$ . (Note: in-k is an accepted abbreviation for in-kip).
- 2) The 216,500 kip elastic strain limit value, that was used to calculate the elastic axial strain energy above, is also used to calculate the plastic axial strain energy. When the columns as a group reach their elastic limit, many will be able to sustain the value of  $A \times F_y$ , i.e. cross sectional area times the yield stress. The thinner walled columns will not. A 3% axial strain limit is commonly assumed for class 1 sections, and lesser proportional amounts for classes 2, 3 and 4. Taking an average between 3 and zero (zero for class 4 since they will buckle locally before reaching yield), we get 1.5% strain. The shortening of a column in a given

story will thus be the height of the column of 149 inches times 0.015 = 2.24 inches. It follows that the axial plastic energy is 216,500 kips x 2.24 inches = 485,000 in-k.

- 3) After the 2.24 inch plastic strain occurs, rather than continuing to squash like a pancake, the columns will deform by forming plastic hinges at the top, bottom and at mid-height within the story and then buckle. The energy dissipation here is calculated in the same manner used in the Bazant model, in which the total rotations summed at the three locations = 2 pi. There will be fully plastic moments for the stockier sections that can maintain  $M_p$  for several degrees of rotation before the bending capacity diminishes. For the less stocky columns (classes 2 and 3)  $M_p$  is initially reached and then degradation sets in. For the class 4 thin-walled columns,  $M_p$  is never reached, but a value of 0.5  $M_p$  is likely. Finally, a scissors shape will occur in all columns of the 97<sup>th</sup> story with the 98<sup>th</sup> floor squashing the space between it and the 97<sup>th</sup> floor slab with a corresponding energy drain of 2,103,000 in-k.

Adding these up, a total energy drain of 552,000 + 485,000 + 2,103,000 = 3,140,000 in-k is realized. However, this is only a part of the energy drain that needs to be considered. Since the lower columns of the upper block would be subject to equal but opposite forces, these columns would also be expected to suffer axial elastic and plastic deformation and buckling. The forces applied to the upper block will, in fact, be exerted on the columns of the 99<sup>th</sup> story, at the bottom of the upper block. With the forces being equal and opposite, the total damage to the structure of the upper block, if calculated, would show an equivalent total energy drain to that occurring in the structure of the lower block. However, here we are only quantifying the energy required to deform and buckle the columns on the 99<sup>th</sup> story, as we did for only the columns on the 97<sup>th</sup> story of the lower block. Since the 99<sup>th</sup> story columns had 93% of the size of the columns on the 97<sup>th</sup> story, they result in 93% of the energy drain found for the columns on the 97<sup>th</sup> story, with the difference being accounted for by stress wave propagation to points further up in the upper block structure.

The total amount of energy dissipation for the columns on both the 97<sup>th</sup> and 99<sup>th</sup> stories is thus 1.93 times that for the 97<sup>th</sup> story and it calculates as 1.93 x 3,140,000 in-k = 6,060,000 in-k.

As shown earlier, the weight of the upper 12 stories plus the roof had a value of 69,303 kips and the velocity determined by the regression analysis, at 1 second into the collapse and just prior to impact, is  $V_1 = 22.81$  ft./s. The kinetic energy of the upper 12 stories plus the roof, dropping a height of 11.44 feet to the 98<sup>th</sup> floor slab below, can be found using the equation  $\frac{1}{2}MV_1^2$ , while also dividing the weight by the acceleration due to gravity to get mass. A value of 6,725,860 in-k is found for the kinetic energy of the upper block, at the time of impact of the 99<sup>th</sup> and 98<sup>th</sup> story floor slabs. The after impact velocity  $V_2$  can be found by subtracting the dissipated energy from the kinetic energy just prior to impact and solving the equation below for velocity.

$$6,725,860 \text{ in-k} - 6,060,000 \text{ in-k} = \frac{1}{2}MV_2^2$$

The value of  $V_2$  is 7.18 ft./s, reflecting a velocity reduction of 15.63 ft./s due to the three calculated energy drains of axial elastic deformation, axial plastic deformation, and plastic hinge buckling of the columns on the 97<sup>th</sup> and 99<sup>th</sup> stories.

## APPENDIX E

### CALCULATION OF VELOCITY REDUCTION DUE TO CONSERVATION OF MOMENTUM

The upper block consists of the 99<sup>th</sup> through 110<sup>th</sup> stories plus the roof with an approximate weight of 69,303 kips, the mass of which we will designate as  $M$  ( $= 2,152$  k-slugs). The measured velocity of the upper block, when it contacts the floor slab of the 98<sup>th</sup> story, was 22.81 ft./s (based on a height between floor slabs of  $h = 11.44$  feet), which we will designate here as  $V_1$ .

If the masses of the 98<sup>th</sup> story columns and floor slab are added to the original mass of the falling upper block, the new mass becomes  $13/12M$ .

A velocity drop will occur due to conservation of momentum and can be found using the equation

$$M \times V_1 = 13/12M \times V_1'$$

As mass drops out of the equation we are left with

$$12/13V_1 = V_1'$$

Knowing  $V_1$  from the actual measurements and solving we find the new velocity  $V_1' = 21.06$  ft./s reflecting a reduction in velocity due to conservation of momentum of 1.75 ft./s.