Analysis of Mass and Potential Energy in the World Trade Center Twin Towers

Gregory H. Urich
B.S. Electrical and Computer Engineering

Abstract

The mass of one of the Twin Towers is calculated based on available data and estimated live loads. The potential energy for one of the Twin Towers is calculated based on the mass of the tower distributed over the various floors. The mass for each floor is established based on the average mass per floor adjusted for differences in mass due to stronger steel structures lower in the tower. All floors including mechanical floors and the basement floors are treated equally with regard to superimposed dead-loads.

Introduction

Many references can be found with different values for the mass of and the amount of potential energy stored in the WTC twin towers. A number of references are shown in Table 1 below. None of these references provide any data or calculation method on which the mass and potential energy are based. The purpose of this paper is to establish a substantiated value for the mass and potential energy of one tower.

Table 1: Different values for mass and potential energy given by references

Source	Mass	Potential Energy
Ashley [7]	500,000 tons	
Bazant and Zhou [6]	≥ 480,000 tons (metric)*	
Hamburger, et al. (FEMA) [4]		4 E+11 J
Tyson [2]	500,000 tons	
Wikipedia [5]	500,000 tons	

** calculated based on mass given for upper part of North Tower = 58 E+6 kg*

Analysis

In the design documentation for WTC1 and WTC2 the structural loads are divided into dead-loads, super-imposed dead-loads, and live-loads. These divisions are also used here.

Dead-loads

Foundation

The mass of the foundation is provides no load on structural components other than itself and contributes a negligible amount to potential energy. The mass of the foundation is nonetheless approximated based on the film footage from the Port Authority of New York and New Jersey.[1] Dimensions are established by comparison to objects of known size, i.e. humans.

The foundation for the core columns was comprised of steel reinforced concrete footers and steel grillages built up out of I-beams. One steel grillage is made up of 17 I-beams with approximate dimensions 0.75m x 0.2m x 2m with a plate thickness around 0.03m. Each grillage also had a base plate for the core column with approximate dimensions 1m x 1m x

0.3m. It is assumed that there is one grillage per core column. Using a density of 7.784 metric tons per cubic meter for the density of A35 steel, the total mass for the grillages is approximately 484 metric tons. Each grillage was placed on a concrete footer with approximate dimensions 2.5m x 2.5m x 2m. Using a density of 2.4 metric tons per cubic meter, the total mass for the concrete footers is approximately 1410 metric tons.

The foundation for the external columns was comprised of a continuous, steel reinforced, concrete footer and base plates ranging from 7 to 9 square feet (approx. 0.74 m^2). The reference for this value is unsure but it is most likely from FEMA or NIST. The thickness of the base plate is unknown but a thickness of 3 cm is assumed. Using a total number of 80 exterior columns (transition to 238 columns at 7th floor), the total mass of the base plates is approximately 14 metric tons. The concrete footer for the external columns had a perimeter of 252 meters. The other dimensions of the footer are unknown but are approximated using 2 meters for depth and 2 meters for width. The total mass for the concrete footer is thus 2420 metric tons.

Table 2: Mass of foundation

Component	Mass (short tons)	Mass (metric tons)
Core steel grillage w/ base plate	534	484
Core concrete footer	1555	1410
External column steel base plates	15	14
External column concrete footer	2670	2420
Total mass foundation	**4774**	**4328**

Structural steel

NIST's value for the mass of steel used in one tower is 100,000 short tons.[3] A simplified approximation based on averaging component dimensions provided by NIST demonstrated that this value is reasonable.

The actual mass of the upper floors is less than the lower floors due to heavier supporting structures lower in the building. FEMA describes a variation in thickness of exterior column plates from 4 inches at the base to ¼ inch in the upper stories.[4] This indicates a ratio of 16 to 1 for structural steel from bottom to top. The mass of the steel can be scaled linearly as a function of floor number from the bottom to the top as follows:

$$m_{steel}(f) = m_{avg} \bullet (-30f + 3710)/1955$$

f is the floor number, m_{avg} is the average mass of steel per floor (= 99,451 tons/116 floors; foundation components are subtracted)

Mass above grade:

$$\sum_{f=7}^{116} m_{avg} \bullet (-30f + 3710)/1955 = 89{,}416 \text{ short tons}$$

Mass below grade:

$$\sum_{f=1}^{6} m_{avg} \bullet (-30f + 3710)/1955 = 10{,}035 \text{ short tons}$$

Table 3: Mass of structural steel per floor in short tons *(Note: floor 116 has been transposed to 110 to correspond to the normal floor numbering. Also, 549 tons has been used for floor 0, i.e. steel in the foundation.)*

floor	mass	floor	mass	floor	mass	floor	mass	floor	mass	floor	mass
110	101	90	364	70	627	50	890	30	1153	10	1416
109	114	89	377	69	640	49	903	29	1167	9	1430
108	127	88	390	68	653	48	917	28	1180	8	1443
107	140	87	403	67	667	47	930	27	1193	7	1456
106	153	86	417	66	680	46	943	26	1206	6	1469
105	167	85	430	65	693	45	956	25	1219	5	1482
104	180	84	443	64	706	44	969	24	1232	4	1495
103	193	83	456	63	719	43	982	23	1245	3	1509
102	206	82	469	62	732	42	995	22	1259	2	1522
101	219	81	482	61	746	41	1009	21	1272	1	1535
100	232	80	496	60	759	40	1022	20	1285	0	1548
99	246	79	509	59	772	39	1035	19	1298	-1	1561
98	259	78	522	58	785	38	1048	18	1311	-2	1574
97	272	77	535	57	798	37	1061	17	1324	-3	1587
96	285	76	548	56	811	36	1074	16	1338	-4	1601
95	298	75	561	55	824	35	1088	15	1351	-5	1614
94	311	74	574	54	838	34	1101	14	1364	-6	549
93	325	73	588	53	851	33	1114	13	1377		
92	338	72	601	52	864	32	1127	12	1390		
91	351	71	614	51	877	31	1140	11	1403		

Concrete floor slabs above grade (Floors 1-110)

Floor slabs outside of the core were constructed primarily of light concrete. The mass of light concrete can be calculated using the floor area outside of the core (approx. 28,225 sq ft), the floor thickness (4 in. [8]), and the density of light concrete (109.3 lb/ft^3).

28,255 sq ft/floor x 0.33 ft x 109.3 lb/ft^3 x 110 floors x 1 ton/2000 lbs = 56,600 short tons

Floor slabs inside the core were constructed primarily of normal concrete. The mass of normal concrete used in these floors can be calculated using the floor area (11,745 sq ft), the floor thickness (5 in. [8]), and the density of normal concrete (150 lb/ft^3).

11,745 sq ft/floor x 0.4167 ft x 150 lb/ft^3 x 110 floors x 1 ton/2000 lbs = 29,400 short tons

Concrete floor slabs below grade (Floors B1-B6)

Floor slabs below grade were constructed primarily of normal concrete. The mass of normal concrete used in these floors can be calculated using the floor area (40,000 sq ft), the floor thickness (8 in. [8]), and the density of normal concrete (150 lb/ft^3).

40,000 sq ft/floor x 0.6666 ft x 150 lb/ft^3 x 6 floors x 1 ton/2000 lbs = 8,700 short tons

Superimposed Dead-loads

Superimposed dead-loads are considered permanent non-varying loads from non-structural components such as wiring, plumbing, heating and cooling aggregates, elevators, etc. Unfortunately the dead loads are very difficult to approximate due to the lack of information about what elements comprised them. Superimposed dead-loads in the WTC towers are considerably higher in the so called mechanical floors. This is however ignored for simplicity and an average superimposed dead-load is approximated and distributed throughout all floors. The design documents give a superimposed dead-load of 8 psf for most floors outside of the core. [8] This value is most likely larger than the actual loads but is used for all floors to take into account the much larger actual loads of the mechanical floors.

Mass of superimposed dead-loads above grade:

40,000 sq ft/floor x 8 lb/ft2 x 110 floors x 1 ton/2000 lbs = 17,600 short tons

Mass of superimposed dead-loads below grade:

40,000 sq ft/floor x 8 lb/ft2 x 6 floors x 1 ton/2000 lbs = 960 short tons

Live-loads

Live-loads are approximated using 1/4 (as used by NIST) the maximum design loads.

Above grade, the most predominate design load outside of the core was 100 lbs/sq ft.[8]

25 lbs/sq ft x 28,255 sq ft/floor x 110 floors x 1 ton/2000 lbs = 38,850 short tons

Above grade, the most predominate design load inside the core was 50 lbs/sq ft. [8]

12.5 lbs/sq ft x 11,745 sq ft/ floor x 110 floors x 1 ton/2000 lbs = 8,075 short tons

Below grade, the most predominate design load inside the core was 500 lbs/sq ft. [8]

125 lbs/sq ft x 40,000 sq ft/ floor x 6 floors x 1 ton/2000 lbs = 15,000 short tons

Total Mass

The total mass is 279,000 short tons or 254,000 metric tons.

Table 4: Mass above grade

Component	Mass (short tons)	Mass (metric tons)
Concrete floor inside core area	29 400	26 671
Concrete floor outside core area	56 600	51 347
Structural steel	89 416	81 117
Live-load inside core	8 075	7 326
Live-load outside core	38 850	35 244
Superimposed dead-load	17 600	15 966
Total mass above grade	**239 941**	**217 671**

Table 5: Mass below grade

Component	Mass (short tons)	Mass (metric tons)
Concrete foundation	4 221	3 829
Concrete floor	8 700	7 893
Structural steel	10 035	9 104
Live-load	15 000	13 608
Superimposed dead-load	960	871
Total mass below grade	**38 916**	**35 304**

Potential Energy

The potential energy (u) due to gravity (close to earth) of any object can be calculated as:

$$u = mgh$$

m = mass, g = acceleration due to gravity, h = height

A reasonable approximation for potential energy relative to ground level (above grade) can be made using:

$$\sum_{f=1}^{110} (m_{non\text{-}steel} + m_{steel}(f)) \bullet 9.8 m/s^2 \bullet (414{,}53 m \bullet f/110) = 398{,}000 \text{ MJ}$$

$m_{non.steel}$ is the average mass (converted to metric) of one floor excluding structural steel, $m_{steel}(f)$ is the value for the mass of steel for a particular floor from Table 3 (converted to metric), g = acceleration due to gravity, f is the floor number, 414,53 is the height of the tower above grade in meters

The sum is from 1 to 110 to include all floors above ground and the roof and their underlying support structure. Potential energy per floor is show in Table 6 below. The total potential energy is 3.98×10^{11} J.

Table 6: Potential energy per floor (above grade)

floor	PE(MJ)	floor	PE(MJ)	floor	PE(MJ)	floor	PE(MJ)	floor	PE(MJ)	floor	PE(MJ)
110	5415	90	5223	70	4680	50	3783	30	2534	10	933
109	5413	89	5205	69	4643	49	3729	29	2463	9	844
108	5411	88	5185	68	4606	48	3674	28	2390	8	753
107	5408	87	5164	67	4568	47	3618	27	2317	7	662
106	5405	86	5143	66	4529	46	3562	26	2242	6	570
105	5400	85	5121	65	4489	45	3504	25	2167	5	477
104	5394	84	5097	64	4448	44	3446	24	2091	4	384
103	5388	83	5073	63	4406	43	3386	23	2014	3	289
102	5380	82	5048	62	4363	42	3326	22	1936	2	194
101	5372	81	5022	61	4320	41	3265	21	1857	1	97
100	5363	80	4996	60	4276	40	3203	20	1778		
99	5353	79	4968	59	4230	39	3140	19	1697		
98	5342	78	4939	58	4184	38	3076	18	1616		
97	5330	77	4910	57	4137	37	3012	17	1534		
96	5318	76	4880	56	4089	36	2946	16	1450		
95	5304	75	4849	55	4040	35	2880	15	1366		
94	5290	74	4817	54	3991	34	2812	14	1281		
93	5275	73	4784	53	3940	33	2744	13	1196		
92	5258	72	4750	52	3889	32	2675	12	1109		
91	5241	71	4715	51	3836	31	2605	11	1021		

Discussion

One difficulty in approximating the potential energy is that the dimensions for core columns are unknown. Since the structural components are stronger (i.e. heavier) lower in the building, it is necessary to know how these components varied over the height of the building. Some dimensions for core box columns given by NIST are not correct. For example, the dimensions "as large as 12 in. by 52 in., comprised of welded plates up to 7 inches thick" must be incorrect. It can be seen from the photographic evidence that the thickest plates are used for the larger dimension of the rectangular box columns. Thus, the width dimension would need to at least 14 inches to accommodate the 7 inch thick plates.

Accuracy of the calculation

Due to certain limitations of available information and also the method of calculation, the values for mass and potential energy are not perfectly accurate. Factors which may affect the accuracy are listed in the below along with estimated effects caused by reasonable deviation.

Factor	Deviation	Effect on mass and PE
A significant part of the floor space inside the core was used for elevator shafts and such so the actual floor space could be reduced.	- 10%	- 1%
Structural steel was mostly below the level of the floor rather than at floor level as used in the calculation.	<< 1% of the height	(PE only) negligible
Mass of structural steel per floor could vary more or less than 93.75% with height.	± 5%	(PE only) < 1%
Value given for steel by NIST could be inaccurate.	± 10%	± 3%
Estimated live-load and superimposed dead-loads could be inaccurate.	± 10%	± 3%
Floors 1-6 were special purpose floors so dead-loads, live-load and superimposed dead-loads are probably inaccurate.	± 10%	< 1%

Conclusion

The calculated mass of one tower is 253,000 metric tons. The total potential energy above grade is 3.98×10^{11} J. This indicates that the value for mass given by Ashley, Bazant and Zhou, and Wikipedia are nearly 80% more than the actual mass of one tower. The value for potential energy given by FEMA is probably correct. It is interesting to note that the mass of the upper part of the North Tower (i.e. above floor 96) given by Bazant and Zhou is nearly three times higher than if calculated by this method.

References

1. Port Authority of New York and New Jersey, "Building the World Trade Center." (1983) http://www.pbs.org/wgbh/amex/newyork/sfeature/sf_building.html

2. Tyson, P., "Towers of Innovation." *PBS/NOVA* http://www.pbs.org/wgbh/nova/wtc/innovation.html

3. Gayle, F.W., et al., "NIST NCSTAR 1-3 Mechanical and Metallurgical Analysis of Structural Steel." *NIST Federal Building and Fire Safety Investigation of the World Trade Center Disaster* http://wtc.nist.gov/reports_october05.htm

4. Hamburger, R., et al., (May 2002) "World Trade Center Building Performance Study, Chapter 2: WTC1 and WTC2." *FEMA* 403 http:/www.fema.gov/rebuild/mat/wtcstudy.shtm

5. Wikipedia, "World Trade Center." *Wikipedia* http://en.wikipedia.org/wiki/World_Trade_Center

6. Bazant, Z.P., Zhou, Y., (in press 9/13/01, Expanded 9/22/01, Appendices 9/28/01) "Why Did the World Trade Center Collapse?—Simple Analysis." *Journal of Engineering Mechanics ASCE*

7. Ashley, S., (October 09, 2001) "When the Twin Towers Fell." *Scientific American*

8. Lew, H.S., Bukowski, R.W., Carino, N.J., "NIST NCSTAR 1-1 Design, Construction, and Maintenance of Structural and Life Safety Systems." *NIST Federal Building and Fire Safety Investigation of the World Trade Center Disaster* http://wtc.nist.gov/reports_october05.htm