I have a response to the paper entitled, “Momentum Transfer Analysis of the Collapse of the Upper Storeys of WTC 1” By Gordon Ross in the Journal of 9/11 Studies. I will only address this one critical point in his paper, buckling, which he does not appear to have even a basic understanding of.

First let me say that the building did not collapse like the pancake theory suggests. In reality it was much more complicated with individual elements being overloaded before the rest of the structure was fully loaded, lateral loads induced from eccentric loading as well as multiple floors of columns failing at the same time. However Ross as well as Bazant and Zhou make these assumptions and I will not challenge them as they make the math easy.

Ross’s paper fails on his knowledge of buckling failure; buckling is the phenomenon which causes a member to bow in compression. However for the reader to understand what causes buckling, a rudimentary knowledge of the engineering stress-strain curve is required. Stress is a measurement of the force based upon the cross-sectional area of the member. Strain is the deformation (unit-less fraction) of that member.

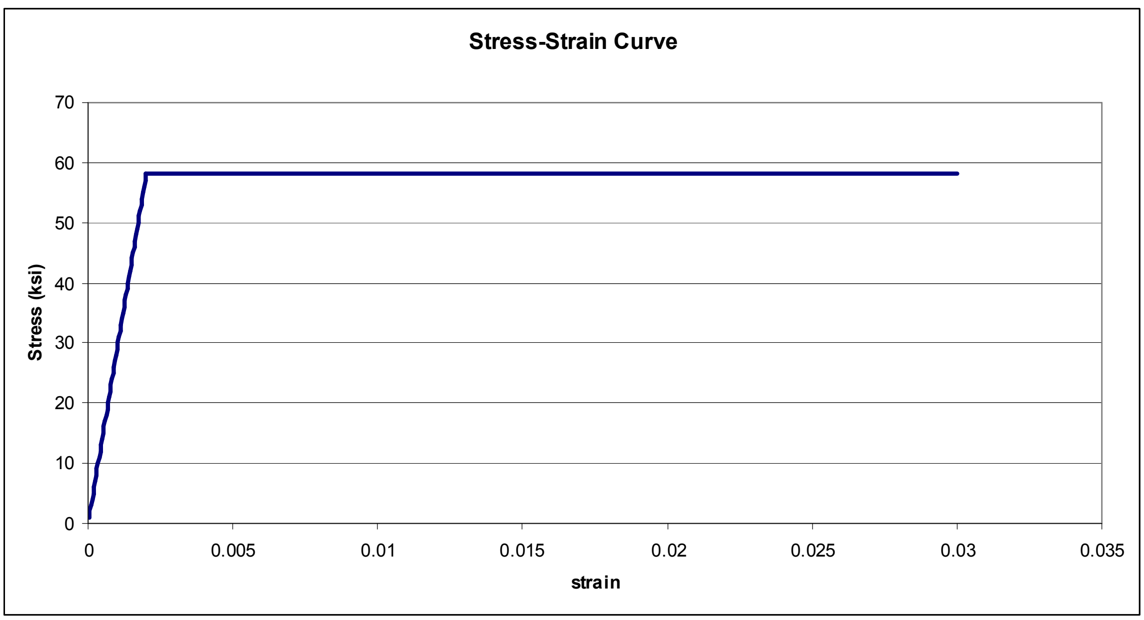

Figure 1 - Stress vs. Strain

Figure 1 is a simplified graph of the stress-strain curve for steel with a yield point of 58ksi, which also occurs at a strain of 0.002 (or 0.2%). The yield stress, Fy, is the place on the graph where the curve becomes horizontal. The slope of the diagonal line from the origin to the yield point is known as the Modulus of Elasticity, E, which is 29000 ksi for structural steel.

The strain energy of a member is a function of the area under the Stress vs. Strain curve. Specifically, it is defined as:

$$W = \int_0^u F(u)du$$

This is nothing more than a fancy way of saying that the energy is the area under the curve of the axial force F(u) as a function of the displacement and. strain (u) curve. There is also a component of strain-energy for the bending (bowing due to buckling) portion of the column failure, however I will not get into it here. The above is just to show what Ross is doing in his calculations because he doesn't really put it forward in understandable terms.

The actual derivation of buckling is non-trivial and I will not attempt to do it here. However, the AISC Manual of Steel Construction Specification has an entire section devoted to it and I will use its equations, only slightly modified to put the equations in terms of stress rather than axial load. First, there are two forms of buckling: inelastic and elastic (Euler). Elastic buckling occurs in very slender elements, Inelastic in short and stubby sections. The slenderness of a column is defined by ratio KL/r.

Where:
K = effective length factor, which will be assumed to be 1.0.
L = height of column between supports (in)
r = radius of gyration (in). (For those who have a knowledge of physics or engineering, r is the square root of the second moment of area, or moment of inertia over the cross sectional area).

When:

$KL/r < 4.71 * \sqrt{E/Fy}$ (Inelastic Buckling)

$\sigma_{cr} = 0.658^{\frac{Fy}{Fe}}$ [AISC 13th E3-2]

Where:

$Fe = \frac{\pi^2 E}{(KL/r)^2}$ [AISC 13th E3-4]

When:

$KL/r > 4.71 * \sqrt{E/Fy}$ (Elastic Buckling)

$\sigma_{cr} = 0.877 * Fe/Fy$ [AISC 13th E3-3]

σ_{cr} is the critical stress in which the column yields and cannot resist any more vertical force. AISC provides a graph showing the relationship between the Design Stress, which is 0.9 times σ_{cr} and slenderness, shown in Figure 2. With a slenderness ratio of 0, the design stress is equal to the yield point times 0.9. The columns at or near the impact in the WTC had a slenderness ratio between 20 and 40. This means that critical stress is actually less than yield stress.

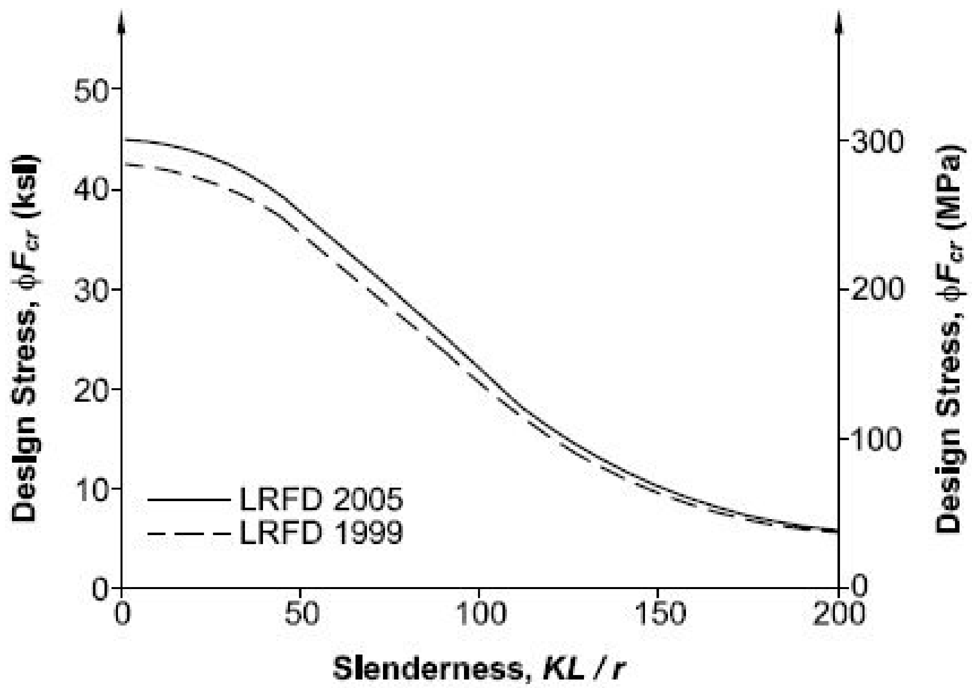

Figure 2 – Slenderness vs. Design Stress.

Buckling always occurs before the yield stress in a compression member. Ross doesn't seem to think so. He describes a compression mode of failure, which buckles at 3% strain, "2/ The shortening phase allows for the same failure load to be applied until the vertical deformation reaches 3% at which point the column begins to form buckle points." This is not correct. At 3% strain the column has all but disintegrated. It does not buckle at this point, it's structurally non-existent.

Ross even acknowledges how important buckling is. A compression member cannot resist any additional vertical load after the critical stress (see figure 2) is reached. In fact, if the load is kept constant, the column will bow until it breaks from bending while resisting less and less axial force. This greatly reduces the strain-energy in that member. He even goes so far as to state this fact in his "Assumptions and Disregards" section. However, he then immediately says that buckling did not occur because the columns were not of sufficient length for Euler, or Elastic, buckling to take place, "Euler calculations show that columns of the dimensions used in the towers would not fail due to buckling over a length of one storey height, but would instead adopt a compressive failure mode." He completely ignores a complete range of buckling with this statement, and with it, completely over-exaggerates the strain energy available in the columns.

Even with knowing how important buckling is, Ross doesn't know what it really is. In the last part on buckling in his "Assumptions and Disregards" section, he states that, "…I have chosen a buckling failure mode as this mode has the lowest energy demand." In reality, he has not picked a buckling failure mode. This is evident in his calculations (which I have other issues with, but will not get in to) where he attempts to take the energy from the full 0.2% to 3% at the yield stress. His calculations show that he has assumed the column to be in pure axial compression without buckling until 3% strain, where a normal member would have ruptured, and then says that he has chosen not to look at the energy from the stress-strain curve after the 3% which doesn't even exist structurally.

Gordon Ross does not have a clear understanding of structural engineering, and this is evident in his paper. His idea of the concept of buckling is incorrect. This in turn has led him to write a paper that completely over-exaggerates the structural capacity of the WTC towers and completely mislead a group of people who depended on him, as a professional engineer, to know what he was talking about. This is not his fault, or the fault of the education system which trained him. He is not a structural engineer; these concepts are not readily available to him. Even I have made mistakes on this concept before. We all make mistakes. This is one that could probably be corrected.

References:

Gordon Ross, Momentum Transfer Analysis of the Collapse of the Upper Storeys of WTC 1 http://www.journalof911studies.com/articles/Journal_5_PTransferRoss.pdf

Bazant and Zhou, Why Did the World Trade Center Collapse?—Simple Analysis1 http://www.civil.northwestern.edu/people/bazant/PDFs/Papers/405.pdf

AISC Manual of Steel Construction, 13th Edition

ASCE Code of Ethics, https://www.asce.org/inside/codeofethics.cfm