Momentum Transfer Analysis of the Collapse of the Upper Storeys of WTC 1

Author:
The author of this work, Gordon Ross, was born in Dundee, Scotland. He holds degrees in both Mechanical and Manufacturing Engineering, graduating from Liverpool John Moores University, in 1984. He can be contacted at gordonjross@yahoo.com.

Summary:

This paper examines the elastic loading and plastic shortening phases of the columns of WTC 1 after impact of the upper 16 storeys of the building upon the lower storeys and its effect on the momentum transfer after the collision. An energy balance is derived showing that there is an energy deficit before completion of the plastic shortening phase that would not allow the collapse to continue under the constraints of this paper.

Introduction:

Previous analysis of the momentum transfer in the collapse of the towers has viewed them as being floors suspended in space and have examined the momentum transfer as a series of elastic or inelastic collisions, which are independent of each other. This type of analysis takes the momentum transfer out of the context given by the other effects of the collisions. This is because this type of analysis assumes that the impacts have an effect upon only the topmost storey of the impacted section. The reality of the situation is that the impacts would have an effect upon several storeys in the lower section and for a valid analysis all of these momentum transfers must be included.

If we assume that the upper section comprising 16 storeys falls under a full gravitational acceleration through a height of one (removed) storey, a distance of 3.7 metres we can calculate that its velocity upon impact will be 8.52 metres per second and have a kinetic energy due to its mass and velocity of 2.105 GJ. (Using the figure of 58000 tonnes as detailed in the report by Bazant & Zhou.[1]) In reality there would be some losses of energy due to residual strength within the failing columns of the removed section, but these are ignored for the purposes of this analysis.

Upon impact with the lower section the falling mass would deliver a force which would grow from zero, up to the failure load of the impacted storey columns, over a finite period of time and distance.

This force would also be felt by the columns below the storey which was first impacted.

Analysis:

The falling upper section with a velocity of no more than 8.5 metres per second at impact would meet resistance from the impacted columns and have as its first task the necessity to load these columns through their elastic range and thereafter through the plastic shortening phase. We shall firstly examine this incremental time period.

Bazant/Zhou [1] show in their analysis that elastic and plastic behaviour of a steel column under a dynamic buckling load can be shown to consist of three distinct phases. These can be shown on a load against vertical deflection graph and consist of an initial elastic phase, a shortening phase and a rapid plastic deformation phase.

1/ The elastic phase shows a linear relationship between load and deflection up to the elastic limit. The load at this point is the failure load and the deflection at the elastic limit for steel is generally 0.2% of the initial length.
2/ The shortening phase allows for the same failure load to be applied until the vertical deformation reaches 3% at which point the column begins to form buckle points.
3/ The third phase shows a rapid decrease in the load requirement to continue deformation, this load necessarily being less than the failure load. This phase lasts until the total vertical deformation equals the original length. In other words the column is bent in two.

To shorten the columns of the first impacted storey by 3%, sufficient to complete the plastic shortening phase, a distance of about 0.111 metres, and allowing a constant speed of 8.5 metres per second, would take a minimum of 0.013 seconds.

The speed of the propagation wave through a medium is given by the general formula for wave propagation

Velocity = Square root (Bulk modulus / Density),

and for structural steel is of the order of 4500 metres per second.
The propagation wave of the impact force would therefore travel a distance of 58.7 metres in a time of 0.013 seconds. This means that during the time taken in the plastic shortening of the impacted columns, the same force would be felt at a minimum distance of 58.7 metres, or approximately 16 storeys, from the impact. These storeys would thus suffer an elastic deflection in response to, and proportional to, the failure load applied at the impacted floor. These deflections would themselves take time and allow the propagation wave to move further downwards again affecting more storeys.

We can estimate the elastic deflection of these 16 storey columns as being in the range 0 to 7mm. The full elastic deflection of a 3.7m column, using the generally accepted figure of 0.2% of its original length is 7.4mm. The columns in the uppermost of these storeys would suffer almost their full elastic deflection since their failure load is similar though slightly greater than that of the first impacted storey. Those storey columns more distant from the impact would be of a larger cross section, requiring higher loads to cause full elastic deflection. Using only half of the maximum elastic deflection, 56mm (16 * 7 / 2), is, again, an assumption in favour of collapse continuation.

The elastic deflection of lower storeys would increase the distance through which the falling section would have to move in order to load the impacted column and complete its 3% plastic shortening. The time taken, again using a constant velocity of 8.5 m/sec would increase to about 0.02 seconds, and thus allow the propagation wave to move through and affect a further 8 storeys.

Because these columns suffer a vertical deflection, the attached floors move downwards and they will therefore have a velocity and momentum.

Energy Losses:

A simple conservation of momentum calculation, ignoring these movements, would have, 16 falling storeys moving at 8.5 m/sec before impact, changing to 17 storeys moving at (8.5 * (16/17)) = 8 m/sec after impact. This does not reflect the fact that a minimum of 24 further storeys will be caused to move downwards at varying speeds.

To estimate and illustrate the further momentum changes we can assume that the storey which is 25 storeys from the impact remains static and the velocity of the 24 affected storeys will vary linearly from the velocity of the falling section to zero.

Momentum before impact = 16 storeys moving at 8.5 m/sec
Momentum after impact = 17 storeys moving at V2 m/sec + 1 storey moving at 23/24*V2 m/sec + 1 storey moving at 22/24*V2m/sec +……+ 1 storey moving at 2/24*V2 m/sec + 1 storey moving at 1/24*V2m/sec 16*8.5 = V2 (17 + 11.5)

V2 = 16 * 8.5 / 28.5 = 4.8 metres per second.

The speed of the upper section would be reduced by the collision from 8.5 m/sec to a speed of less than 4.8 m/sec rather than the 8 m/sec derived from a momentum calculation which does not include this factor. Note also that this reduction in speed would again give more time for the propagation wave to travel downwards through the tower columns and allow that more and more storeys are so affected.

The kinetic energy of the falling section would be similarly affected, but because of the velocity squared relationship, the reduction in kinetic energy would be more pronounced.

K. E. of falling section before impact = 16 floors moving at (8.5 m/sec)

K. E. of falling section after impact = 17 floors moving at (4.8 m/sec)

Percentage loss of K.E. = 1-(17 * 4.8/ (16 *8.5) * 100% = 66%

This is an underestimation of the energy loss, since the deceleration would allow more time for travel of the propagation wave and so allow more floors to be affected but even this shows an energy absorption of some 66% of the total kinetic energy of the falling section.

Energy Balance:

Since there was only some 2.1 GJ available at the point of impact of the first collision, a loss of 66% would reduce this figure to 714 MJ.
The kinetic energy would be augmented by potential energy released in the further downward movement of the falling mass and if we assume that this falls through the full distance of the 3%

shortening phase of the impacted floor and the elastic deflection of the lower storeys, then the additional potential energy is

58*10* g * (0.111 + .056) = + 95MJ.

The strain energy consumed by the impacted storey columns in the elastic phase and plastic shortening phase can be calculated using the failure load. The failure load used throughout this analysis is derived using the mass above the impact, 58 000 tonnes, and a safety factor of 4. Examination of the column geometry with reference to the Euler equations show that this is an underestimation both of the failure load and the distance over which it would have to act before failure, and this gives a gross assumption in favour of collapse continuation. A factor of 0.029 is included to reflect the load profile over the 3% plastic shortening phase. The load profile exhibits a linear rise from zero to failure load at 0.2% of the length, followed by a constant failure load over the next 2.8% of the length.

Plastic strain energy:

58*10kg*4*g*3.7m*0.029= -244MJ.

A similar though slightly smaller figure would be required for the first impacting storey in the upper falling section. Because this storey carried a lower load, 15 storeys, than the impacted storey, 17 storeys, its designed capabilities would be proportionately smaller.
Using this knowledge an estimation can be made that the energy consumed by this storey would be,

(244 MJ * 15 / 17) = -215MJ .

The elastic response of the lower storey columns within their elastic range would make further demands on the energy available by absorption of energy in the form of strain energy. This can be estimated, using a safety factor of 4, a mass of 58000 tonnes, a distance of 0.056metres, and a factor of 0.5 to reflect the load profile

58*10kg*4*g*0.056metres*0.5= -64MJ.

The downward movement of these floors in response to the impact will release additional potential energy due to their compression and using the same deflections as above and a value for mass proportionate to the number of storeys, this will release

58*10kg * 24/16 * g * 0.056metres / 2 = + 24 MJ.

Further energy losses are evident in an analysis of the compression of storeys within the upper falling section. These storeys manufactured from columns with a smaller cross section than those at the impact, would be unable to withstand the failure load present at the impact front and would suffer plastic deformation beyond their elastic limit, but for simplicity, it is assumed that they suffer only their full elastic deflection. This is another large assumption in favour of collapse continuation.

The total deflection would be 15 storeys multiplied by the elastic deflection of 7.4mm, and strain energy consumed can be estimated as,

15*7.4*10*4*58*10*g/2= -126MJ.

Movement of the storeys within the upper section will release additional potential energy due to their compression and consequent movement. It is likely that this energy would manifest itself as failures within the upper section, but has nevertheless been added as an energy available for collapse continuation. The uppermost storey will move downwards by 15 times the elastic deflection whereas the lowest will remain static, both in relation to the impact point, giving additional potential energy as, 15*0.0074*58*10*g/2= +32MJ

A considerable amount of energy would be required to pulverise the concrete into the fine dust which was evident from the photographic and other evidence. To quantify this energy it is necessary to use the fracture energy value, but this has a variable value dependent on, among other factors, the size of the concrete piece, and its constituents, most notably, aggregate size. There is no typical value.

In order to assess the energy consumed I will refer to the work of Dr. Frank Greening [2]. It should be noted that Dr. Greening, like Dr. Bazant, does not, as yet, support the contention that the tower collapse was caused by anything other than the damage caused by aircraft impact and subsequent and consequent fires.

The tower, using Dr. Greening's figures, contained approximately 50000 tonnes of concrete, and the assumption is made that only 10% of this was pulverised to a size of 60 micrometres. One kilogram of concrete at this particle size will have a surface area of 67 m^2. We can now use Dr. Greening's figure for concrete fracture energy of 100J/m^2 to show that the energy requirement for one floor would be 50*10^6kg / 110floors * 67m^2 * 100J/m^2 * 10% = - 304 MJ.

It may be considered unlikely that a low velocity impact would expend large energies on pulverisation of materials, and this is more likely in later stages of the collapse. However, the large expulsions of dust were visually evident at early stages of the collapse.

Energy Summary:
The energy balance can be summarised as
Energy available;
    Kinetic energy 2105MJ
    Potential energy Additional downward movement 95MJ
    Compression of impacting section 32MJ
    Compression of impacted section 24MJ
    Total Energy available 2256MJ
Energy required;
    Momentum losses 1389MJ
    Plastic strain energy in lower impacted storey 244MJ
    Plastic strain energy in upper impacted storey 215MJ
    Elastic strain energy in lower storeys 64MJ
    Elastic strain energy in upper storeys 126MJ
    Pulverisation of concrete on impacting floor 304MJ
    Pulverisation of concrete on impacted floor 304MJ
    Total Energy required 2646MJ
Minimum Energy Deficit -390MJ

Conclusion:

The energy balance of the collapse moves into deficit during the plastic shortening phase of the first impacted columns showing that there would be insufficient energy available from

the released potential energy of the upper section to satisfy all of the energy demands of the collision. The analysis shows that despite the assumptions made in favour of collapse continuation, vertical movement of the falling section would be arrested prior to completion of the 3% shortening phase of the impacted columns, and within 0.02 secondsafter impact.

**A collapse driven only by gravity would not continue to progress beyond that point.
The analysis shows that the energies expended during the time period of the plastic shortening of the first storey height of the vertical columns is sufficient to exhaust the energy of the falling section and thereby arrest collapse. This however is not the full extent of the plastic strain energy demand which exists. The next immediate task for the falling mass to continue in its descent would be the plastic shortening within the remainder of the buckle length. As has already been stated a buckling failure mode has a minimum length over which it can act and in the case of the towers would be several storey lengths.
Each additional storey length involved in the buckle would add a further demand of about 450MJ for a further downward movement of 0.111metres. This also shows that collapse arrest is not dependent upon an expenditure of energy in concrete pulverisation, since even if this expenditure were disregarded the input energy would be exhausted during plastic shortening of the second storeys affected.**

The analysis can be extrapolated to show that the energy expended within the plastic shortening phase of a six storey buckle would ensure that a fall by the upper section through two storeys under full gravitational acceleration would also be resisted at an early stage. A similar response would be elicited from an opposed three or more storey drop delivering the same levels of energy at impact. It can be further envisaged that a collapse initiated by a fall through a greater number of storeys, would be either arrested or significantly and noticably slowed when regard is taken for energy demands both in the fall by the upper section, and by inclusion of demands identified but not quantified in this article. It should also be noted that this analysis examines only the energy levels required up to a point in time during the plastic shortening phase. Energy demands which involve further phases of the collapse mechanism, such as buckling of beams and disassociation of end connections, spandrel plates and floor connections are further massive energy demands which must then be satisfied.

Assumptions and disregards :

A buckling failure is notable because of the characteristic reduction in load required to continue failure after yield is reached, being distinct from a compressive failure where the load to continue failure after yield is substantially greater than the yield load, and will reach a maximum at the Ultimate Load. In the immediate time period after impact the force applied by the falling section will manifest as such a compressive load. Euler calculations show that columns of the dimensions used in the towers would not fail due to buckling over a length of one storey height, but would instead adopt a compressive failure mode. The load would increase to yield levels, and due to the work hardening which would be present here but not in a buckle failure, thereafter increase towards the Ultimate Load level and this would manifest as plastic compression or shortening, until such time as enough length of column to satisfy the minimum length requirements of buckling, had been

exposed to the load. The tower columns when viewed individually had dimensions which would dictate a minimum length for buckling of three or more storey heights. When the bracing of the spandrel plates and corners of the perimeter columns, and the horizontal and diagonal bracing is taken into regard the minimum buckling length would extend over many storey heights. At this point the load would continue to manifest as plastic compression or shortening, but also as a tendency to buckle the column, rather than continue in compression failure. The energy profile would thereafter become that of a buckle failure.The analysis would be justified in using the greater energy demand characteristics of a compressive failure mode for the first instances of the collapse, but I have chosen a buckling failure mode as this mode has the lowest energy demand.

The assumption of constant velocity of the falling mass ignores the immediate deceleration which would be felt by the falling mass. As an example, if we asumed that the velocity was halved over the distance covered in this analysis the time would be extended by one third, giving more time for the energy to dissipate to more remote points.
The analysis assumes a linear distribution in the elastic deflections and velocities of the affected floors during calculation of the momentum transfer and elastic strain energy. Since most of the column sections involved would have undergone almost their full elastic deflection, this treatment underestimates the energy demands within those calculations.

Only a second iteration has been used to show the number of floors taking part in the momentum and velocity changes of the collision. A full iteration would give about 30 storeys, and allowing that the falling mass was decelerated to half of its original velocity would allow time for the propagation to extend loading to more than 40 storeys below the impact. My assumptions have the affect of reducing the number of storeys which take part. This together with the assumption that only a portion of the elastic deflection will apply underestimates the energy requirements of this task.

The characteristic of steel to show an increase in Young's modulus in response to an impact load is acknowledged as a further energy demand but is not quantified.
It should be understood that the energy losses referred to as momentum losses cannot be re-employed as strain energy or in the energy required to pulverise the floors, thereby reducing the total energy demand. These energy transfers would exist irrespective of the state of repair of the floors after collision and would exhibit as heat in the impacted materials.

The kinetic energy being considered is that of the impacting mass of the falling section. There is kinetic energy in the now moving lower storeys but this has been lost by the impacting mass. The only source of energy which is available to the falling mass is potential energy and unless that energy is released by collapse of further columns the falling mass will come to a halt. As the propagation wave continues to load columns further down the tower the energy will spread through lower storeys as elastic strain energy which is recoverable, unlike plastic strain energy. As the upper section decelerates, the force which it is capable of exerting will reduce, and the elastic deflection will reduce in response. As this drops the elastic strain energy previously absorbed by the lower storeys will convert back to potential energy. In other words it will unload, or bounce. The towers were best

characterised as being a series of springs and dampers, being struck with a large but relatively slow moving and less substantial series of springs and dampers.

Damage in this analysis aside from the storey removed in order to initiate collapse is limited to the damage to the two storeys which impacted each other, and even this was not sufficient to move the impacted columns through the plastic shortening phase and into the rapid plastic phase which is characterised and accompanied by the onset of buckle points. It should be noted that this concentrates the energy of the impact. In reality several of those storeys nearest to point of impact and especially those with columns of lighter cross section in the upper falling section would each suffer a portion of that damage. This would further serve to dissipate the energy at points remote from the collapse front.

An initiation mechanism involving a total and instantaneous loss of all load bearing ability on one storey, sufficient to cause a 3.7m drop under full gravitational acceleration followed by a neat impact is not credible. This is presented to show the relative sizes of the energies involved. This analysis underestimates the energy demands by using a constant value of velocity, equal to the velocity at impact,
8.5 m/sec. This is an assumption made in favour of collapse continuation.
This analysis also assumes that each storey had the same mass. The effect that this assumption has, is to underestimate the energy losses at collision. No account has been taken of the mass which falls outside the tower perimeter, and most notably neither of the expulsion of large amounts of dust early in the collapse, nor of the energy requirement to cause these masses to move outside the perimeter.

This analysis takes no regard of the energy consumed in damage caused to spandrel plates or other structural elements, nor disconnection of the floor to column connections, crushing of floor contents, nor of any other energies expended. No account is taken of any strain energy consumption during the initial fall through the height of one full storey, though this would be a substantial proportion of the initial energy input.

References:
[1] Journal of Engineering Mechanics ASCE, 9/13/01, Expanded 9/22/01, Appendices 9/28/01) Why Did the World Trade Center Collapse?—Simple Analysis By Zdenek P. Bazant1, Fellow ASCE, and Yong Zhou